Hyperbolically embedded subgroups have been defined by Dahmani-Guirardel-Osin and they provide a common perspective on (relatively) hyperbolic groups, mapping class groups, $Out(F_n)$, CAT(0) groups and many others. I will sketch how to extend a quasi-cocycle on a hyperbolically embedded subgroup $H$ to a quasi-cocycle on the ambient group $G$. Also, I will discuss how some of those extended quasi-cocycles (of dimension 2 and higher) "contain" the information that $H$ is hyperbolically embedded in $G$. Joint with Roberto Frigerio and Maria Beatrice Pozzetti.