The notion of antichain catching appeared in the Foreman-Magidor-Shelah paper on Martin’s Maximum, and was used extensively in Woodin’s proofs of the presaturation of various stationary tower forcings. For normal ideals $I$ and $J$, let us say that $J$ catches $I$ (and write $\text{catch}(J, I)$) iff $J$ has sufficiently large support, the $J$-positive sets project onto the $I$-positive sets in a certain canonical manner (as ideals), and whenever $G \subset (J^+, \subset)$ is generic then the projection of $G$ is generic for $(I^+, \subset)$. Certain instances of $\text{catch}(J, I)$ are equivalent to saturation of $I$ (namely when $J$ is the conditional club filter relative to $I$; see Foreman’s chapter in Handbook of Set Theory). But in general the statement:

“there exists a $J$ such that $\text{catch}(J, I)$”

is strictly weaker than saturation of $I$ and strictly stronger than precipitousness of $I$. I will discuss this result and others from some joint work with Martin Zeman; I will also discuss some joint work with Matteo Viale which made use of related notions.