A gradient Ricci soliton consists of a complete Riemannian manifold $(M, g)$ and a smooth function $u$ on $M$ which satisfy the equation
\[
\text{Ric}(g) + \text{Hess}_g(u) + \frac{\epsilon}{2} g = 0.
\]
While there is a vast literature on gradient Ricci solitons, there are very few examples, except in the Kähler case and in the non-compact homogeneous case (when the solitons must be of expanding type).

In this talk, I will discuss some properties of gradient Ricci solitons which are special to the cohomogeneity one situation and which may be pertinent to their construction. The talk is based on joint work with A. Dancer and S. Hall.