By a result of Kedra and Pinsonnault, we know that the topology of groups of symplectomorphisms of symplectic 4-manifolds is complicated in general. However, in all known (very specific) examples, the rational cohomology rings of symplectomorphism groups are finitely generated. In this paper, we compute the rational homotopy Lie algebra of symplectomorphism groups of the 3-point blow-up of the projective plane (with an arbitrary symplectic form) and show that in some cases, depending on the sizes of the blow-ups, it is infinite dimensional. Moreover, we explain how the topology is generated by the toric structures one can put on the manifold.