In the first part of this talk, we consider general rank minimization problems. We first show that a class of matrix optimization problems can be solved as lower dimensional vector optimization problems. As a consequence, we establish that a class of rank minimization problems have closed form solutions. Using this result, we then propose penalty decomposition (PD) methods for general rank minimization problems in which each subproblem is solved by a block coordinate descend method. Under some suitable assumptions, we show that any accumulation point of the sequence generated by our method when applied to the rank constrained minimization problem is a stationary point of a nonlinear reformulation of the problem. In the second part, we consider general $l_0$-norm minimization problems. We first reformulate the $l_0$-norm constrained problem as an equivalent rank minimization problem and then apply the above PD method to solve the latter problem. Further, by utilizing the special structures, we obtain a PD method that only involves vector operations. Under some suitable assumptions, we establish that any accumulation point of the sequence generated by the PD method satisfies a first-order optimality condition that is generally stronger than one natural optimality condition. Finally, we test the performance of our PD methods on matrix completion, nearest low-rank correlation matrix, sparse logistic regression and sparse inverse covariance selection problems. The computational results demonstrate that our methods generally outperform the existing methods in terms of solution quality and/or speed.