The Lie algebra of the group of diffeomorphisms of the circle $\text{Diff}(S^1)$ is the set of smooth vector fields tangent to the circle. Although this Lie algebra has a complexification, by a theorem of Lempert, there is no group whose Lie algebra is this complexification. As part of his sketch of a mathematical formulation of conformal field theory, G. Segal introduced a semigroup of annuli with boundary parametrizations or “riggings”, which in some sense is the complexification of $\text{Diff}(S^1)$.

By enlarging the class of boundary parameterizations, we show that this semigroup is in fact a discrete quotient of the Teichmüller space of an annulus. In particular this introduces a complex structure on the semigroup, in which multiplication is holomorphic. If time allows we will explore some further connections with geometric function theory.

Joint work with David Radnell.