Let $k$ be an algebraically closed field of characteristic 2, and let $W$ be the ring of infinite Witt vectors over $k$. Suppose $G$ is a finite group, and $B$ is a block of $kG$ with dihedral defect group $D$ which is Morita equivalent to the principal 2-modular block of a finite simple group. We determine the universal deformation ring $R(G, V)$ for every $kG$-module $V$ which belongs to $B$ and has stable endomorphism ring $k$. It follows that $R(G, V)$ is always isomorphic to a subquotient ring of $W^D$. Moreover, we obtain an infinite series of examples of universal deformation rings which are not complete intersections.