Is it useful to think of orbifolds as stacks?

I have been told by a number of people that one should think of orbifolds as Deligne–Mumford stacks. I have also been warned that “it is difficult to explain what stacks are and it is even more difficult to explain why it’s the right way to think about orbifolds.” I will report on what happened when I tried to come to grips with orbifolds as stacks.