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Subspaces of Hilbert space

The relative position of two 1-dimensional subspaces of Hilbert space is expressed fully by a single number in $[0\ 1]$. But suppose we study subspaces P spanned by orthonormal $p_1, p_2, \ldots, p_k$ and Q spanned by orthonormal $q_1, q_2, \ldots, q_k$ for higher k. To express the relative position of P and Q fully requires an unordered k-tuple of numbers; this theory has been understood since the 1880s and generalized to infinite-dimensional subspaces. In contrast, it is known that the relative position of three k-dimensional subspaces can not be expressed fully by a manageable invariant.

Nevertheless the manifold of k-subspaces can be studied to advantage. Here is a typical example of a question which can be asked of three subspaces and may have illuminating answers: Given subspaces P, Q, R, is P closer to Q than R is? This talk gives a modern way of dealing with such questions.