A hypergraph is linear if every two hyperedges share one point at most. We investigate the following question. What is the maximum possible number of edges in a linear $k$-uniform hypergraph $H$ with $n$ vertices which does not contain a given linear $k$-uniform hypergraph $G$ as a subgraph? We show applications to geometry and number theory.