Let $\alpha$ be a string over $\mathbb{Z}_q$, where $q = 2^d$. The $j$-th elementary symmetric function evaluated at $\alpha$ is denoted $e_j(\alpha)$. We study the cardinalities $S_q(m; \tau_1, \tau_2, \ldots, \tau_t)$ of the set of length $m$ strings for which $e_i(\alpha) = \tau_i$. The profile $k(\alpha) = \langle k_1, k_2, \ldots, k_{q-1} \rangle$ of a string $\alpha$ is the sequence of frequencies with which each letter occurs. The profile of $\alpha$ determines $e_j(\alpha)$, and hence $S_q$. Let $h_n: \mathbb{Z}_{2^{n+d-1}} ightarrow \mathbb{Z}_2[z] \mod z^{2^n}$ be the map that takes $k(\alpha) \mod 2^{n+d-1}$ to the polynomial $1 + e_1(\alpha)z + e_2(\alpha)z^2 + \cdots + e_{2^{n-1}}(\alpha)z^{2^n-1}$. We show that $h_n$ is a group homomorphism and establish necessary conditions for membership in the kernel for fixed $d$. The kernel is determined for $d = 2, 3$. The range of $h_n$ is described for $d = 2$. These results are used to “efficiently” compute $S_q(m; \tau_1, \tau_2, \ldots, \tau_t)$.

This is joint research with Bob Miers at UVic.