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Asymptotics of largest and smallest components in combinatorial structures

We study the extremal size of components in decomposable combinatorial structures. We have in mind combinatorial objects such as permutations that decompose into cycles, graphs into connected components, polynomials over finite fields into irreducible factors and so on. The probability that the size of the smallest components of combinatorial objects of size n be at least m is explained by the Buchstab function, as shown by Panario and Richmond (2001) using the Flajolet–Odlyzko singularity analysis method. In this talk, we adapt Buchstab's recursive arguments for integers to combinatorial objects. We study the probability of connectedness for structures of size n when all components have size at least m. In the border between almost certain connectedness and almost certain disconnectedness, we encounter a generalized Buchstab function.

For largest components, using singularity analysis, Gourdon (1996) has studied the probability that structures of size n have all components of size at most m. We also give a recursive argument for the probability of connectedness for structures of size n when all components have size at most m. In this case, our results are given in terms of a generalized Dickman function. The Dickman function appears when studying the largest prime factor of a random integer between 1 and n.

We apply these results to several combinatorial structures such as permutations, polynomials over finite fields, labelled 2-regular graphs, functional digraphs, trees (unrooted and rooted case), labelled and unlabelled graphs, Achiral trees, and k-point labelled stars.

Based on joint works with: Ed Bender, Atefeh Mashatan and Bruce Richmond (JCTA 2004) and Mohamed Omar, Bruce Richmond and Jacki Whitely (to appear in Algorithmica).