Critical values for the moduli space of symplectic balls in a rational 4-manifold

(joint work with Martin Pinsonnault)

We compute the rational homotopy type of the space of symplectic embeddings of the standard ball $B^4(c) \subset \mathbb{R}^4$ into 4-dimensional rational symplectic manifolds of the form $M_\lambda = (S^2 \times S^2, (1 + \lambda)\omega_0 \oplus \omega_0)$ where $\omega_0$ is the area form on the sphere with total area 1 and $\lambda$ belongs to the interval $[0,1]$. We show that, when $\lambda$ is zero, this space retracts to the space of symplectic frames, for any value of $c$. However, for any given $\lambda > 0$, the rational homotopy type of that space changes as $c$ crosses the critical parameter $c_{\text{crit}} = \lambda$, which is the difference of areas between the two $S^2$ factors. We prove moreover that the full homotopy type of that space change only at that value, i.e. the restriction map between these spaces is a homotopy equivalence as long as these values of $c$ remain either below or above that critical value. The same methods apply as well to other rational 4-manifolds like $\mathbb{CP}^2$ or the topologically non-trivial $S^2$-fibration over $S^2$. 