Schubert calculus is the study of “positivity properties” in several rings associated to the homogeneous spaces, notably equivariant cohomology and equivariant K-theory. We show how one can get new and old formulas for the structure constants in these rings. We introduce new operators whose coefficients compute Schubert structure constants (in a manifestly polynomial, but not positive, way), resulting in a formula that generalizes the positive AJS/Billey formula. Our proof involves Bott-Samelson manifolds and in particular, the cohomology basis dual to the homology basis of classes of sub-Bott-Samelson manifolds.