Finite-dimensional linear flows are among the simplest dynamical systems imaginable. When exactly do two such flows have the same phase portrait, up to a change of coordinates? If the change of coordinates and rescaling of time are differentiable, this question is easy to answer; otherwise, it is much more delicate. Nevertheless, complete answers have been given, independently, by N. Kuiper and N. Ladis in the 1970s. Although the ensuing topological classification of linear flows is easy to state, the basic ideas behind it have not found their way into textbooks. This talk attempts to rectify this. A new, completely elementary proof of the classification theorem is presented which only requires basic linear algebra. As an aside, several inaccuracies in the classical arguments will be corrected. (Joint work with A. Wynne.)