Representation Theory
Théorie des représentations
(Org: **Thomas Creutzig** and/et **Nicolas Guay** (Alberta))

MURRAY BREMNER, University of Saskatchewan
Classification of regular parametrized one-relation operads

Jean-Louis Loday introduced a class of symmetric operads generated by one bilinear operation subject to one relation making each left-normed product of three elements equal to a linear combination of right-normed products:

$$(a_1a_2)a_3 = \sum_{\sigma \in S_3} x_\sigma a_{\sigma(1)}(a_{\sigma(2)}a_{\sigma(3)});$$

such an operad is called a parametrized one-relation operad. For a particular choice of parameters $\{x_\sigma\}$, this operad is said to be regular if each of its components is the regular representation of the symmetric group; equivalently, the corresponding free algebra on a vector space V is, as a graded vector space, isomorphic to the tensor algebra of V. We classify, over an algebraically closed field of characteristic zero, all regular parametrized one-relation operads. In fact, we prove that each such operad is isomorphic to one of the following five operads: the left-nilpotent operad defined by the identity $((a_1a_2)a_3) = 0$, the associative operad, the Leibniz operad, the dual Leibniz (Zinbiel) operad, and the Poisson operad. Our computational methods combine linear algebra over polynomial rings, representation theory of the symmetric group, and Gröbner bases for determinantal ideals and their radicals.

GERALD CLIFF, University of Alberta
PBW, canonical, and crystal bases for quantized enveloping algebras

Lusztig has defined PBW bases P for the minus part of the quantized enveloping algebra of a semi-simple complex Lie algebra. It is known that these PBW bases have a crystal structure in the sense of Kashiwara, but it is not known, in general, how to define Kashiwara's crystal operators on P. We show how this can be done for types A, B, C, and D, using tableaux of Kashiwara-Nakashima. (In type A, these are the usual semi-standard Young tableaux.) This is new for types B and C.

CLIFTON CUNNINGHAM, University of Calgary
An example of Vogan's geometric description of Arthur packets for p-adic groups

In 1992, David Vogan conjectured how one might use techniques from microlocal geometry to study Arthur packets of irreducible admissible representations of connected reductive groups over local fields. Shortly after, Adams, Barbasch and Vogan proved this conjecture for reductive groups over Real numbers. By contrast, Vogan's conjecture remains open for reductive groups over p-adic fields. In this talk we give a precise statement of Vogan's conjecture for Arthur packets of admissible representations of p-adic groups and the stable distributions attached to them by Arthur's work; this version of the conjecture makes no mention of microlocal geometry. Then we give an example of this conjecture, involving 15 representations of p-adic $\mathrm{SO}(7)$, both split and anisotropic forms, and confirm the conjecture in this case. This is joint work with Andrew Fiori, James Mracek, Ahmed Moussaoui and Bin Xu.

YUN GAO, York University
Quantized GIM algebras

Generalized intersection matrix Lie algebra was introduced by Slodowy. It is known that a GIM Lie algebra is a fixed point of an involution of Kac-Moody Lie algebra for a double-sized generalized Cartan Matrix. We define a quantum version for simply-laced GIM Lie algebras and obtain a quantum analog of the above result.

VINCENT X. GENEST, Massachusetts Institute of Technology
Multifold tensor product modules of $su_q(1,1)$, trigonometric superintegable systems, and multivariate q-special functions

In this talk, I will explain how to construct separated wavefunctions for q-analogs of second-order superintegrable systems in any dimension. The construction is based on the decomposition of multifold tensor product modules of the quantum algebra $su_q(1,1)$ in irreducible components using multivariate q-special functions of q-Hahn or q-Jacobi type as generalized recoupling coefficients.

ALLEN HERMAN, University of Regina
Unitary groups over p-adic integers - the ramified case

When L is a quadratic extension of a p-adic number field K, the Galois automorphism of L acts nontrivially on the ring of integers $\mathcal{O}_L$. Composing with the transpose induces an involution $*$ on $M_n(\mathcal{O}_L)$. The resulting unitary group $U_n(\mathcal{O}_L) = \{X : X^{-1} = X^*\}$ satisfies the congruence subgroup property, which means any continuous finite-dimensional representation factors through a congruence subgroup. This reduces the study of representations of these groups to the study of representations of unitary groups over the finite local rings $R = \mathcal{O}/\mathcal{P}^N$. Of particular interest is the description of irreducible constituents of the Weil representation of $U_n(R)$ in this situation. Previous work of Gow and Szechtman treated the case where p is odd and L/K is unramified. Recently we have calculated the orders of these unitary groups $U_n(R)$ when p is odd and L/K is ramified. We have determined irreducible constituents of the Weil representation and calculated the degrees of these characters when the level N is even, using tools from character theory and hermitian geometry over local rings. This is joint work with Fernando Szechtman, James Cruikshank, and Rachael Quinlan.

MIKHAIL KOTCHETOV, Memorial University
Graded-simple algebras and modules via the loop construction

The construction of (twisted) loop and multiloop algebras plays an important role in the theory of infinite-dimensional Lie algebras. Given a grading by $\mathbb{Z}/m\mathbb{Z}$ on a semisimple Lie algebra, the loop construction produces a $\mathbb{Z}$-graded infinite-dimensional Lie algebra.

This was generalized by Allison, Berman, Faulkner and Pianzola to arbitrary nonassociative algebras and arbitrary quotients of abelian groups. In view of their results, the recent classification of gradings by arbitrary abelian groups on finite-dimensional simple Lie algebras (over an algebraically closed field of charactersitic zero) yields a classification of finite-dimensional graded-simple Lie algebras.

Mazorchuk and Zhao have recently applied the loop construction to modules. In this talk, we will show how this leads to a classification of finite-dimensional graded-simple modules over simple Lie algebras with a grading. This is joint work with Alberto Elduque.

MICHAEL LAU, Université Laval
Harish-Chandra modules for current algebras

Current algebras are Lie algebras of regular maps from an affine variety to a finite-dimensional simple Lie algebra. We will discuss the classification of simple weight modules (with finite dimensional weight spaces) for current algebras.

ANDREW LINSHAW, University of Denver
Orbifolds and cosets via invariant theory

The orbifold and coset constructions are standard ways to create new vertex algebras from old ones. It is believed that orbifolds and cosets will inherit nice properties such as strong finite generation, C_2-cofiniteness, and rationality, but few general results of this kind are known. I will discuss how these problems can be studied systematically using ideas from classical invariant theory. This is based on joint work with T. Creutzig.

MONICA NEVINS, University of Ottawa
On archetypes and an inertial Langlands correspondence

We summarize recent work by Henniart, Latham, Nadimpalli and others towards an inertial local Langlands correspondence, and present some preliminary new results. A key step is to replace so-called types, which often encode construction data for representations but are defined on a variety of compact open subgroups, with the closely-related notion of an archetype, which lives on a maximal compact open subgroup. There is a growing body of results relating archetypes to the restrictions to inertia of corresponding L-parameters in an nice way.

ATHENA NGUYEN, University of British Columbia
Local Multiplicity One Theorem for GL_n and L-functions

In 1966, Andre Weil remarked that the results of the local theory in Tate's thesis can be viewed as stating that the space $\mathrm{Hom}_{k^\times}(C_c^\infty(k), \chi)$ is one-dimensional for every smooth character χ of $k^\times$. Moreover, the origin of the generator of $\mathrm{Hom}_{k^\times}(C_c^\infty(k), \chi)$ differs depending on the L-function of χ. Weil, then, asked for a generalization of such a result to $\mathrm{GL}_n(k)$. A partial answer has been provided by Godement-Jacquet, and Moeglin, Vignéras, Waldspurger using zeta-integrals. In this talk, I will revisit this problem and discuss the connection between L-functions and the local multiplicity one theorem for $\mathrm{GL}_2(k)$ in particular, and some partial results for $\mathrm{GL}_n(k)$.

MATTHEW RUPERT, -University of Alberta
Logarithmic Hopf Link Invariants for $\overline{U}_q^H(\mathfrak{sl}(2))$

Little is known about Vertex Operator Algebras (VOAs) which are neither C_2-cofinite nor rational, and most of the work on such VOAs has been focused on specific examples such as the Singlet. It is thought that the representation categories for the Singlet and the unrolled restricted quantum group associated to $\mathfrak{sl}(2)$, $\overline{U}_q^H(\mathfrak{sl}(2))$, are closely related. In this talk I will provide an overview of the relationships between these categories and present results on the representation category of $\overline{U}_q^H(\mathfrak{sl}(2))$. In particular, I will demonstrate an efficient method for computing open Hopf links and Alexander invariants colored with projective modules via families of deformable modules.

YVAN SAINT-AUBIN, Université de Montréal
The category of Temperley-Lieb algebras and their fusion product

The Temperley-Lieb algebras $\mathsf{TL}_n(\beta)$ appear in several chapters of mathematics and physics. In the latter, one particular element of a $\mathsf{TL}_n(\beta)$ captures the Boltzmann weights of several statistical models. The family of algebras $\mathsf{TL}_n(\beta), n \geq 1$, was cast into a category by Graham and Lehrer (1998). Independently Read and Saleur (2007) introduced a fusion product on the modules over these algebras, that is an operation (a functor) that maps two modules into a third one. These modules are in general over distinct algebras of the Temperley-Lieb family.

We show that the category of the Temperley-Lieb algebras is braided and that this braiding can be extended naturally to a category of modules over the family for the product introduced by Read and Saleur.

Joint work with J. Belletête.

LUC VINET, CRM, Université de Montréal
A Superintegrable Model on the 3-sphere with Reflections and the Rank 2 Bannai-Ito Algebra

I shall present a quantum superintegrable model on the 3-sphere with reflections. Its symmetry algebra will be identified as the rank-two Bannai-Ito algebra. It will shown that the Hamiltonian can be constructed from the tensor product of four irreducible representations of the superalgebra osp(1,2) and that its superintegrability is naturally understood in that setting. The exact separated solutions will be obtained through the Fisher decomposition and a Cauchy-Kovalevskaia extension.

Based on work done in collaboration with H. De Bie (Ghent), V. X. Genest (MIT), J.-M. Lemay (CRM).

CURTIS WENDLANDT, University of Alberta

Finite-dimensional representations of twisted Yangians of types B,C and D

Recently, a new family of twisted Yangians have been introduced which are in one-to-one correspondence with symmetric pairs of types B, C and D. Similar to the untwisted Yangians, these new quantum algebras possess many elegant properties. For instance, it is possible to study their representation theory using a highest weight approach. The goal of this talk is to present some of the first results in that direction, with emphasis on the finite-dimensional irreducible modules. In particular, we will use the notion of a highest weight module to obtain a classification of finite-dimensional irreducible modules for some of these new twisted Yangians. This is joint work with N. Guay and V. Regelskis.

KAIMING ZHAO, Wilfrid Laurier University

Simple W_n^+-modules from Weyl modules and gl_n-modules

For a simple module P over the Weyl algebra K_n^+ and a simple module M over gl_n. Using Shen's monomorphism we make $P \otimes M$ into a module over the Witt algebra W_n^+. I will give the necessary and sufficient conditions for the W_n^+-module $P \otimes M$ to be simple.