Generalized Gradient Flows of Nonlocal Interaction Energies

In this talk, I will consider a generalized notion of gradient flow, i.e., curves of maximal slope, for nonconvex nonlocal interaction energies \( E(\mu) = \int_{\mathbb{R}^d \times \mathbb{R}^d} K(x - y) \, d\mu(x) d\mu(y) \) defined via a pairwise singular interaction potential \( K \) in the power-law form over the space of probability measures with bounded density endowed with the 2-Wasserstein metric. These energies play an important role in models of collective behaviour of multi-agent systems, biological swarming, molecular self-assembly. In particular, I will show that these curves of maximal slopes can be obtained as limits as \( \epsilon \to 0 \) of well-understood gradient flows of semi-convex energies \( E_\epsilon \) defined by a regularization of the interaction potential \( K \). This will also provide a first step in understanding the connection between the gradient flows of unregularized and noncovex interaction energies \( E \) and the aggregation equation

\[
\mu_t - \nabla \cdot (\mu(\nabla K \ast \mu)) = 0
\]

via a singular perturbation approach. This is joint work with Katy Craig (UCLA).