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**Optimization and Nonlinear Analysis**  
**Optimisation et analyse non linéaire**  
(Org: **Mohamed Tawhid** (Thompson Rivers University))

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**AFRAH ABDOU**, King Abdulaziz University

*On Mann's Method with Viscosity for Nonexpansive and Nonspreading Mappings in Hilbert Spaces*

In the setting of Hilbert spaces, inspired by Iemoto and Takahashi (2009), we study a Mann's method with viscosity to approximate strongly (common) fixed points of a nonexpansive mapping and a nonspreading mapping. A crucial tool in our results is the nonspreading-average type mapping.

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**HEINZ BAUSCHKE**, University of British Columbia

*The Douglas-Rachford algorithm: inconsistency and finite convergence*

The Douglas-Rachford algorithm is a classical optimization method for finding minimizers of the sum of two convex (possibly nonsmooth) functions. When specialized to indicator functions of closed convex sets, it will find a point in the intersection of the two sets comprising the associated convex feasibility problem.

In this talk, I will report on two recent developments concerning the Douglas-Rachford algorithm: (1) If the sets are affine but possibly nonintersecting subspaces, then DRA actually converges to the point nearest to the starting point in the "generalized" intersection. (2) In the affine-polyhedral case, finite convergence occurs under a Slater-type constraint qualification.

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**HICHEM BEN-EL-MECHAIEKH**, Brock University, Ontario, Canada

*Systems of Nonlinear Inequalities Without Convexity: A Topological Fixed Point Approach*

The talk discusses the solvability of systems of constrained nonlinear inequalities without convexity, including generalized variational inequalities. The main tools consist of new fixed point results for set-valued maps with non-convex values defined on special classes of neighborhood retracts of interest in optimization.

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**JEIN-SHAN CHEN**, National Taiwan Normal University

*Discovery of new complementarity functions for NCP and SOCCP*

It is well known that complementarity functions play an important role in dealing with complementarity problems. In this talk, we propose a few new classes of complementarity functions for nonlinear complementarity problems and second-order cone complementarity problems. The constructions of such new complementarity functions are based on discrete generalization which is a novel idea. Surprisingly, the new families of complementarity functions possess continuous differentiability even though they are discrete-oriented extensions. This feature enables that many methods like Newton method can be employed directly for solving nonlinear complementarity problems and second-order cone complementarity problems. This is a new discovery to the literature and we believe that such new complementarity functions can also be used in many other contexts.

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**MONICA COJOCARU**, University of Guelph

*Bifurcations in constrained dynamics*

The questions of existence and classification of bifurcation points in a projected dynamics (given by an ODE on a closed convex set arising usually from optimization problems) was studied by various authors so far using either degree theory applied to a variational inequality problem, or using differential inclusions, as introduced by Filipov.

In our most recent work, we encountered equilibrium problems, such as non-cooperative games, where certain parameters induce a change in their associated constrained dynamics; such changes could be appearance or disappearance of periodic cycles, or changes in the number of equilibrium points, or changes in the behaviour of nearby orbits.

While in the classical theory of smooth dynamical systems, these types of changes are clearly studied using mostly the Jacobian of the involved vector field of an ODE, in a constrained dynamics, the non-smoothness of the right hand side poses problems when we are trying to follow the classical approach.

We show in this talk that a known mathematical problem called an evolutionary variational inequality (currently used often in time-dependent equilibrium problem formulations) lends itself as a potential straight forward tool in answering the question of existence of bifurcation values for parameters in a constrained dynamics. We give criteria and examples of how bifurcation questions can be tackled in this context.

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**KIYOKO FURUYA**, Ochanomizu University, Tokyo Jpn  
*On Formally Self-Adjoint Schrödinger operators with real potential*

In this talk we shall introduce our latest paper and related to it. The aim of our paper is to construct a family of unique solutions to Schrödinger evolution equation. A Schrödinger operator, (not self-adjoint but) formally self-adjoint, generates a (not unitary but) contraction semigroup. Our class of potentials  $U$  in Schrödinger equation is wide enough : the real measurable potential  $U$  should be locally essentially bounded except a closed set of measure zero. To prove this we construct an abstract theory of convergence of  $C_0$ -semigroup in Hilbert space. A bounded  $C_0$ -semigroup is not necessarily equi-continuous with respect to the weak topology but equi-continuous with respect to the locally convex topology. In order to apply Hille-Yosida or Trotter-Kato Theorem, the equi-continuity of semigroups is necessary.

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**MOHAMED TAWHID**, Thompson Rivers University  
*Nonlinear Conjugate Gradient Methods for the Output Feedback Pole Assignment Problem*

The nonlinear conjugate gradient methods are very effective iterative methods for solving large-scale optimization problems. Indeed, these methods are widely used to obtain the numerical solution of the optimal control problems arising in various applications such as engineering and

finance, especially for solving large-scale problems. In this article, we propose three nonlinear conjugate gradient methods for solving the output and state feedback pole assignment problems. Also, we establish the global convergence of the proposed algorithms under standard assumptions. Moreover, we extend these methods in order to tackle the output feedback pole assignment problem for decentralized control systems.

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**HENRY WOLKOWICZ**, University of Waterloo  
*Noisy sensor network localization: robust facial reduction and the Pareto frontier*

We consider the localization problem in sensor networks where the inter-sensor distance measurements are inaccurate and incomplete. In this paper, we present two novel algorithms for large-scale sensor localization based on semidefinite programming relaxations. Emerging exoscale networks lead to semidefinite relaxations that are prohibitively large for interior-point methods to handle. Both of our methods are able to efficiently solve the semidefinite programming relaxations without the use of interior-point methods. Our first method works by relating cliques in the graph of the problem to faces of the positive semidefinite cone, allowing us to devise a combinatorial algorithm for the localization problem that is provably robust and parallelizable. Our second algorithm is a first order method for maximizing the trace—a popular low-rank inducing regularizer—in a robust formulation of the localization problem. Namely, we consider the related much easier problem where the trace objective and the robust constraint are interchanged. We provide numerical experiments on large-scale sensor network localization problems illustrating the development. – (work with Dmitriy Drusvyatskiy, Nathan Krislock, Yuen-Lam Voronin)

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**ZILI WU**, Xi'an Jiaotong-Liverpool University  
*Characterization of Weaker Sharp Solutions of a Variational Inequality Problem without Gap Functions*

For a variational inequality problem, its solution set is usually characterized with relevant gap functions if the set is weakly sharp. We characterize the solution set for a more general case. Without any information of gap functions, we further characterize the weak sharpness of the set. In particular, we present an equivalent statement for a pseudomonotone<sup>+</sup> mapping so that the weak sharpness of the set of relevant VIP is of simpler characterization.