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Maps with Memory

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Let $f : X \to X$ be a map. We want to consider a process, which is not a map, and represents situation when $f$ on each step uses not only current information but also some information from the past. We define for current state $x_n$ and $0 < \alpha < 1$:

$$x_{n+1} = f(\alpha x_n + (1 - \alpha)x_{n-1}).$$

We are interested in something we could call an "invariant measure" of the process. We consider ergodic averages

$$A_f(x_0, x_{-1}) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} f(x_i).$$

They are related to ergodic averages of the map $G : X \times X \to X \times X$ defined by

$$G(x, y) = (y, f(\alpha y + (1 - \alpha)x)).$$

We considered the example where $f : [0, 1] \to [0, 1]$ is the tent map. Computer experiments suggest that $G$ behaves in very different manners depending on $\alpha$. We conjecture:

For $0 < \alpha < 1/2$ map $G$ preserves absolutely continuous invariant measure.

For $\alpha = 1/2$ every point of upper half of the square $(y + x \geq 1)$ has period 3 (except the fixed point $(2/3, 2/3)$). Every other point (except $(0, 0)$) eventually enters the upper triangle.

For $1/2 < \alpha < 3/4$ point $2/3, 2/3$ is a global attractor for map $G$.

For $\alpha = 3/4$ every point of the interval $x + y = 4/3$ has period 2 (except the fixed point $(2/3, 2/3)$). Every other point (except $(0, 0)$) is attracted to this interval.

For $3/4 < \alpha < 1$ map $G$ preserves an SRB measure which is not absolutely continuous (supported on an uncountable union of straight intervals).