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Convergent finite difference solvers for the Monge-Ampère equation with Optimal Transportation boundary conditions

The elliptic Monge-Ampère equation is a fully nonlinear Partial Differential Equation which originated in geometric surface theory, and has been applied in dynamic meteorology, elasticity, geometric optics, image processing and image registration. Solutions can be singular, in which case standard numerical approaches fail.

We build a finite difference solver for the Monge-Ampère equation, which converges to the unique viscosity solution of the equation. Regularity results are used to select a priori between a stable, provably convergent monotone discretization and an accurate finite difference discretization. The resulting nonlinear equations are then solved by Newton's method.

Computational results in two and three dimensions validate the claims of accuracy and solution speed. A computational example is presented which demonstrates the necessity of the use of the monotone scheme near singularities.