Discrete and Combinatorial Geometry
Géométrie discrète et combinatoire
(Org: **Ted Bisztriczky** (Calgary) and/et **Wendy Finbow-Singh** (SMU))

LEAH BERMAN, University of Alaska, Fairbanks
Movable 4-Configurations are Plentiful

A geometric k-configuration is a set of points and lines in the plane such that every point lies on k lines and every line passes through k points. Although it is easy to construct symmetric 3-configurations that are "movable"—that is, there is a continuous family of realizations of the same combinatorial configuration which retains the same type of geometric symmetry—finding movable 4-configurations has been more challenging. Indeed, researchers in the field thought that there were no movable 4-configurations that retained geometric symmetry until the first discovery of such a configuration, in 2006; to date, only two basic constructions are known. This talk will discuss preliminary results describing a new construction technique that produces a large variety of chirally symmetric, reasonably small, movable 4-configurations.

ROBERT DAWSON, St Mary's University
Monotone Spreads and Chebyshev Sets in Hyperspaces

A *Chebyshev set* in a metric space is one with the "nearest neighbor" property: every point in the space has a unique nearest neighbor in the set. In $\mathbf{R}^n$ with the usual norm this is more or less the same thing as convexity; with the "taxicab metric" neither implies the other. Here we study Chebyshev sets in metric hyperspaces.

A *monotone arc* in a metric hyperspace of convex bodies is one in which the support functions change in a "translation-like" way along the arc. In certain hyperspaces, closed monotone arcs have the Chebyshev property.

A *monotone spread* is a set of bodies such that every pair of bodies in the spread is joined within the spread by a monotone arc belonging to the set - it's a "convexity-like" property. In the hyperspaces considered in the previous paragraph, monotone spreads also have the Chebyshev property.

This talk will mix this in with a famous theorem of J. Frank Adams, some convex algebraic geometry, and some algebraic topology, and come up with some surprising and hopefully interesting results.

ROBERT ERDAHL, Queens

WENDY FINBOW-SINGH, Saint Mary's University
Modified Spheres

A long standing problem in rigidity theory is to characterize the graphs which are isostatic (rigid and independent) in 3-space. The results of Cauchy (1813), Dehn (1916), and Alexandrov (1950) give one important class: the triangulated convex spheres. In 2012, Whiteley and I provided a second class of 3-dimensional isostatic frameworks, the block and hole polyhedra, along with methods to verify generic rigidity. These methods are based on tracking when a larger framework can be derived from a known small example using vertex splitting, an operation known to take a minimally generically rigid framework to a new minimally generically rigid framework with one more vertex. In this talk we use these methods to investigate more general frameworks: triangulated spheres with an edge removed and an added crossbeam connecting some two non-adjacent vertices.

JIM LAWRENCE, George Mason University
Some polytopes with vertex-transitive symmetry

With each binary matroid it is possible to associate a polytope having vertex-transitive symmetry. We describe this correspondence. For a large collection of these polytopes, the facial structure can be made explicit.

BARRY MONSON, University of New Brunswick
Finite Polytopes have Finite Regular Covers

Every polygon is combinatorially regular, but most 3-polytopes are not. For example, the five Platonic solids are the only convex regular polyhedra. On the other hand, it has been known for some time (and is not so hard to prove) that every convex 3-polytope, or more generally, map on a compact surface, has a regular cover, although here we must venture into the domain of abstract regular polytopes. What about higher dimensions? Recently, Egon Schulte and I proved the natural, but far from obvious, result that every finite abstract d-polytope has a finite regular cover.

MICHAEL MOSSINGHOFF, Davidson College
Sporadic Reinhardt polygons

A *Reinhardt polygon* is a convex n-gon that is optimal in three different geometric optimization problems: it has maximal perimeter relative to its diameter, maximal width relative to its diameter, and maximal width relative to its perimeter. Many Reinhardt polygons exhibit a particular periodic structure, and these are well understood. However, for certain values of n, such as $n = 30$ and $n = 42$, some *sporadic* Reinhardt polygons also occur. We characterize the integers n for which sporadic Reinhardt polygons with n sides exist, and investigate the number of such polygons, relative to the number of periodic ones with n sides. This is joint work with Kevin Hare.

DEBORAH OLIVEROS, Instituto de Matemáticas UNAM
Colourful and fractional (p,q)-theorems.

In this talk, we present some interesting generalizations of the (p, q)-problem of Hadwiger and Debrunner for families of convex sets, in particular some colourful and fractional versions in the sense of Bárány and Lovasz.

This is a joint work with I. Bárány, F. Fodor and L. Montejano.

PATRICIA RIBEIRO, EST Setúbal, Polytechnical Institute of Setúbal
Towards the Standard Spherical Tiling

An isometric folding is a map that sends piecewise geodesic segments into piecewise geodesic segments of the same length. It is known, since 1989, that any non-trivial isometric folding of the euclidian plane is deformable into the standard planar folding $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (x, |y|)$. However, the correspondent situation on the sphere remains an open question.

Related to spherical isometric foldings are spherical f-tilings, that is, edge to edge decompositions of the sphere by geodesic polygons, such that all vertices are of even valency and both sums of alternate angles, around any vertex, are π. The relation between these two sets of objects comes from the fact that the set of singularities of any non trivial isometric folding ia a spherical f-tiling.

As expected, the problem of isometric folding deformations gives rise to a similar problem on spherical f-tiling deformations. More precisely, is any spherical f-tiling deformable into the standard tiling (f-tiling whose underline graph is a great circle)?

Here, we provide a way to deforme into the standard f-tiling, each one of the dihedral f-tilings of the sphere whose prototiles are two non congruent isosceles spherical triangles in a particular way of adjacency.

This is a joint work with Professor Ana Breda.

VALERIU SOLTAN, George Mason University
Convex hypersurfaces with plane quadric sections

The talk will provide a survey of existing results and open problems on characteristic properties of convex qudric hypersurfaces in n dimensions in terms of plane quadric sections.

CSABA TOTH, University of Calgary and CSU Northridge
On the total perimeter of convex bodies in a container

For two convex bodies, C and D, consider a packing S of n positive homothets of C contained in D. We estimate the total perimeter of the bodies in S, denoted per(S), in terms of n. When all homothets of C touch the boundary of the container D, we show that either per$(S) = O(\log n)$ or per$(S) = O(1)$, depending on how C and D "fit together," and these bounds are the best possible apart from the constant factors. Specifically, we establish an optimal bound per$(S) = O(\log n)$ unless D is a convex polygon and every side of D is parallel to a corresponding segment on the boundary of C (for short, D is *parallel to* C). When D is parallel to C but the homothets of C may lie anywhere in D, we show that per$(S) = O((1+\text{esc}(S)) \log n/ \log\log n)$, where esc$(S)$ denotes the total distance of the bodies in S from the boundary of D. Apart from the constant factor, this bound is also the best possible. (Joint work with Adrian Dumitrescu)

ASIA IVIĆ WEISS, York University
Polytopes derived from cubic tessellations

We consider 3- and 4-polytopes arising from a regular tessellation of euclidean space by taking its quotients with a fixed-point-free group of its isometries.

GORDON WILLIAMS, University of Alaska Fairbanks
Monodromy Groups of Belt Polyhedra

The monodromy group of a polyhedron corresponds to its minimal regular cover. Recent work in the study of abstract polytopes has shown that the monodromy group has an interesting role in the study and investigation of the combinatorial structure of abstract polytopes. However, the collection of well understood examples of structural studies of the monodromy group for abstract polytopes is still quite small.

In this talk we will discuss the preliminary results of an investigation into the structure of the monodromy groups for several infinite families of polyhedra formed by attaching symmetric belts of polygonal faces to a pair of parallel n-gons. For example, the n-prism is formed by attaching a belt of squares to two parallel n-gons. Of particular interest is the way in which each of the infinite families corresponds to a parameterized family of abelian extensions of the automorphism group of a regular polyhedron.

DEPING YE, Memorial University of Newfoundland
On the monotone properties of general affine surfaces under the Steiner symmetrization

Affine surface areas are fundamental in (affine) convex geometry and play key roles in many problems such as affine isoperimetric inequalities. The study of affine surface areas has a long history and went back to Blaschke in 1923.

Recently, general affine surface areas were introduced by Ludwig. Such general affine surface areas can involve nonhomogenous (convex and/or concave) functions, and have many nice properties, such as, affine invariance, upper-semicontinuity (or lower-simicontinuity), and valuation. Related affine isoperimetric inequalities have been proved by Ludwig via the Blaschke-Santalo inequality.

In this talk, I will introduce the concept of general affine surface areas, and prove the monotone properties of general affine surfaces under the Steiner symmetrization. As a byproduct, we provide another proof of the affine isoperimetric inequalities for general affine surface areas without assuming the Blaschke-Santalo inequality. Hence, the centroid condition (required by the Blaschke-Santalo inequality) can be removed.