In the 1980s, Guillemin and Sternberg constructed an integrable system on the coadjoint orbits of $U(n)$. This is a geometric analogue of the classical Gelfand-Zetlin bases for irreducible representations. A complexified version was later discovered by Kostant and Wallach on the Poisson manifold $\mathfrak{gl}(n)^*$ in 2006. I will describe an infinite-dimensional analogue of this system on coadjoint orbits of $GL(\infty)$ and discuss some of the associated (infinite) Poisson geometry. This talk is based on joint work with Mark Colarusso.