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Decomposition of level-1 representations of $D_{4}^{(1)}$ with respect to its subalgebra $G_{2}^{(1)}$ in the spinor construction

In Contemp Math, Vol. 121, Feingold, Frenkel and Ries gave a spinor construction of the vertex operator para-algebra $V = V^{0} \oplus V^{1} \oplus V^{2} \oplus V^{3}$, whose summands are 4 level-1 irreps of the affine Kac-Moody algebra $D_{4}^{(1)}$. The triality group $S_{3} = \langle \sigma, \tau \mid \sigma^{3} = 1 = \tau^{2}, \tau \sigma \tau = \sigma^{-1} \rangle$ in $\text{Aut}(V)$ was constructed, preserving $V^{0}$ and permuting $V^{i}$, $i = 1, 2, 3$. $V$ is $\frac{1}{2}\mathbb{Z}$-graded and $V^{i,n}$ denotes the $n$-graded subspace of $V^{i}$. Vertex operators $Y(v, z)$ for $v \in V^{0}$ represent $D_{4}^{(1)}$ on $V$, while those for which $\sigma(v) = v$ represent $G_{2}^{(1)}$. $V$ decomposes into the direct sum of $G_{2}^{(1)}$ irreps by a two-step process, first decomposing with respect to the intermediate algebra $B_{3}^{(1)}$ represented by $Y(v, z)$ for $\tau(v) = v$. There are three vertex operators, $Y(\omega_{D_{4}}, z)$, $Y(\omega_{B_{3}}, z)$, $Y(\omega_{G_{2}}, z)$, each representing the Virasoro algebra given by the Sugawara constructions from the three algebras. These give two coset Virasoro constructions, $Y(\omega_{D_{4}} - \omega_{B_{3}}, z)$ and $Y(\omega_{B_{3}} - \omega_{G_{2}}, z)$, with central charges $1/2$ and $7/10$, respectively, the first commuting with $B_{3}^{(1)}$, the second commuting with $G_{2}^{(1)}$, and each commuting with the other. This gives the space of highest weight vectors for $G_{2}^{(1)}$ in $V$ as tensor products of irreducible Virasoro modules $L(1/2, h_{1}) \otimes L(7/10, h_{2})$.

This dissertation research of my student, Quincy Loney, explicitly constructs these coset Virasoro operators, and uses them to study the decomposition of $V$ with respect to $G_{2}^{(1)}$. This work provides a spinor construction of the $c = 7/10$ Virasoro modules inside $V$, and provides a vertex operator algebra naturally associated with the basic module for $G_{2}^{(1)}$. 