We classify uniform (vertex-transitive) polyhedral maps on surfaces of non-negative Euler characteristic and show that all of them have at most 10 orbits on flags. Furthermore we show that they are all quotients of maps obtained by certain operations applied to regular tessellations on the sphere and on the Euclidean plane.