Leavitt path algebras are a natural generalization of the Leavitt algebras, which are a class of algebras introduced by Leavitt in 1962. For a directed graph $E$, the Leavitt path algebra $L_K(E)$ of $E$ with coefficients in $K$ has received much recent attention both from algebraists and analysts over the last decade. So far, some of the algebraic properties of Leavitt path algebras have been investigated, including primitivity, simplicity and being Noetherian. We explicitly describe two-sided ideals in Leavitt path algebras associated with a row-finite graph. Our main result is that any two-sided ideal $I$ of a Leavitt path algebra associated with a row-finite graph is generated by elements of the form $v + \sum_{i=1}^{n} \lambda_i g^i$, where $g$ is a cycle based at vertex $v$. We use this result to show that a Leavitt path algebra is two-sided Noetherian if and only if the ascending chain condition holds for hereditary and saturated closures of the subsets of the vertices of the row-finite graph $E$. Moreover, we show that this result can be used to unify and simplify many known results for row-finite Leavitt path algebras.