Large cross sections through sets in Euclidean spaces

We consider sets $A \subseteq \mathbb{R}^n \times \mathbb{R}$ which are large either in the sense that their complement either does not contain a Borel set of positive measure or does not contain a nonmeager Borel set. We survey theorems and open problems concerning the existence for such a large set $A$ of cross sections of the form $f \cap A$, where $f : \mathbb{R}^n \to \mathbb{R}$, which are large relative to $f$ (in a suitable sense) and where $f$ is as nice as possible, for example constant (as in Fubini’s theorem) or continuous or smooth or entire.