**Nonlinear Dynamics and Applications**
**Dynamique non linéaire et ses applications**
(Org: **Gail Wolkowicz** (McMaster), **Yuan Yuan** (Memorial) and/et **Xiaoqiang Zhao** (Memorial))

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**JACQUES BELAIR**, Université de Montréal
*Delays and stability in pharmacodynamic modeling*

Modeling of pharmacokinetic and pharmacodynamic processes (PK/PD) is of utmost interest in the development of drugs: quantitative characterisation of the time course of drug concentration in administration context is essential for approval by regulatory authorities. We present recent attempts at a proper mathematical representation of these time courses in the development of cancer drugs. In particular, the role of distributed time delays, either explicitly stated or implicitly introduced in the behavior of solutions, will be discussed. Stability of stationary solutions and possible oscillatory solutions (emerging from Hopf bifurcations) will be presented.

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**ELENA BRAVERMAN**, University of Calgary, 2500 University Drive NW, Calgary, AB, T2N 1N4
*Break of Chaos in Dynamical Systems: Scalar and Spatial Models*

It is well known that as the intrinsic growth rate $r$ increases, the logistic and the Ricker map exhibit an irreversible period doubling route to chaos. If a constant positive perturbation to a Ricker model is introduced, then there is a break of chaos giving birth to a two-cycle (usually through a series of period-halving bifurcations). We study this phenomenon for various models and also in a spatial setting where each cell of the lattice is influenced by its nearest neighbours only. Chaos is not observed for $r$ large enough, and the spatial Ricker model with a uniform positive perturbation can incorporate cells that experience two-cycle oscillations with stable dynamics at some locations. Other models and configurations are considered, for example, one-directional invasions.

This is a joint work with my M.Sc. student Jeff Haroutunian.

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**GUIHONG FAN**, York University, Mathematics/Statistics Ross N520, York University, 4700 Keele Street, Toronto, ON, M3J 1P3
*Hopf Bifurcation in a Delayed Predator Prey Model in the Chemostat*

We study the dynamics of the predator-prey model in the chemostat when a discrete delay is introduced to model the time between the capture of the prey and its conversion to biomass. Holling type I response functions is chosen so that no oscillatory behavior is possible in the associated system when there is no delay. It is shown that as the parameter modelling the delay is varied Hopf bifurcation can occur.

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**ABBA GUMEL**, University of Manitoba
*Mathematics of Chlamydia Transmission Dynamics*

Chlamydia, caused by the bacterium Chlamydia trachomatis, is the commonest sexually-transmitted bacterial disease in European countries and the United States. The annual burden of the disease and its consequences in the USA is estimated at $2 billion. If undetected and treated, the disease could inflict irreversible complications, such as chronic pelvic pain, infertility in females and potentially fatal ectopic pregnancy. The talk will address the problem of the transmission dynamics of chlamydia trachomatis in a population using a mathematical model. The population level impact of various treatment strategies, based on the use of suitable antibiotics, will be discussed.

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**MUDASSAR IMRAN**, McMaster
*Pharmacology of Antibiotic Treatment*

We derive models of the effects of periodic, discrete dosing or constant dosing of antibiotics on a bacterial population whose growth is checked by nutrient-limitation and possibly by host defenses. Mathematically rigorous results providing sufficient conditions for treatment success, i.e., the elimination of the bacteria, as well as for treatment failure, are obtained. Our models can exhibit bi-stability where the infection-free state and an infection-state are locally stable when antibiotic dosing is marginal. In this case, treatment success may occur only for sub-threshold level infections.

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**MAJID JABERI-DOURAKI**, University of New Brunswick
*Dynamics of a higher order non-autonomous difference equation*

We study the non-autonomous difference equation

$$X_{n+1} = F(n, X_n, X_{n-k}) - H(n, X_n), \quad n \geq 0,$$

where $X_{-k}, \ldots, X_{-1}, X_0$ are positive real numbers, $F(n, X_n, X_{n-k})$ is represented by a delay rational difference equation, and $H$ denotes the removal function of individuals during each time period as harvesting or functional responses. In particular, the periodic behavior, the attractivity of solutions and the stability of solutions are discussed in detail. The impact of seasonality on the behavior of solutions is also considered.

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**LUJU LIU**, Xi'an Jiaotong University and Memorial University of Newfoundland
*A tuberculosis model with migration*

A tuberculosis (TB) model with migrant workers is proposed and investigated. The basic reproduction number $\mathcal{R}_0$ is defined. TB disease always dies out and the disease-free equilibrium is globally asymptotically attractive if $\mathcal{R}_0 < 1$; while TB disease uniformly persists in the population and there is at least one endemic equilibrium if $\mathcal{R}_0 > 1$. Furthermore, if the immigration rates of migrant workers from villages to towns or cities and infectious migrant workers from towns or cities to villages are very small, there exists exactly one global attractive endemic equilibrium if $\mathcal{R}_0 > 1$. Numerical simulations indicate that there is only one global attractive endemic equilibrium if $\mathcal{R}_0 > 1$ and the spread of TB may be reduced if effective actions are taken.

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**JUNLING MA**, Department of Mathematics and Statistics, University of Victoria
*Disease spread on contact networks*

Contact network models realistic paths of disease transmission in a population. Network epidemic models describe the spread of diseases on contact networks. Many population dynamics models approximating the intrinsically stochastic disease spread processes have been developed and studied since the introduction of a network SIS model by Pastor-Satorras and Vespignani (2001). But these models have their limitations in modeling the neighboring structure of the network, and thus generally over estimate the exponential growth rate of the epidemic. The basic reproduction number derived from these models are thus not realistic. In this talk, a network SIS model that captures the neighborhood structure of the underlying network is introduced. This model precisely predicts both the growth rate and the equilibrium of the mean dynamics of the corresponding stochastic process. The basic reproduction number can then be computed from this new model.

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**TUFAIL MALIK**, Mount Allison University, Sackville, NB
*A Model of Photoacclimation in Phytoplankton with Optimal Resource Allocation*

We develop a model of the energy budget of phytoplankton using a feedback process to regulate the amount of light-harvesting and carbon-synthesis apparatus and incorporate evolutionary optimization to identify resource allocation strategies, photosynthetic and growth rates. We derive the commonly used saturating photosynthetic-irradiance curves, showing how they are related to our mechanistic parameters. An analytical and numerical comparison of the performance of specialists—grown at specific irradiance—is presented. A specialist is also compared with a generalist, where the latter is characterized as capable of promptly adjusting to the prevailing irradiance.

**JONATHAN MARTIN**, University of Alberta
*Curve Evolutions and Forest Fires*

We present a curve expansion model for forest fire spread in a heterogeneous environment, in the form of a coupled system of hyperbolic partial differential equations. This model extends the mathematical model employed in the Canadian Forest Service's PROMETHEUS fire simulator software package. We will present the derivation of our model, then discuss some analytical results which concern fire perimeter evolutions in anistropic and homogeneous media.

**CONNELL MCCLUSKEY**, Wilfrid Laurier University, Waterloo, Ontario
*Global Stability for Epidemic Models with Delay*

Many ODE models of disease spread have been shown to exhibit the traditional threshold behaviour: *if* $R_0 < 1$*, then the disease-free equilibrium is globally asymptotically stable (GAS); if* $R_0 > 1$*, then the endemic equilibrium is GAS*. Recently, there has been good progress on this through the use of a Lyapunov function that Goh first used for ecological models.

Separately, there have been many functional differential equation (FDE) models of disease spread that have used delay to account for vector transmission, or have used integrals to account for infection-age structure. Earlier results on the global dynamics of these models introduce restrictions on the parameters which seem to be an artifact of the method.

I will discuss recent work that uses Goh-type Lyapunov functionals to resolve the global dynamics for some of these FDE models, achieving the traditional threshold result.

**CHUNHUA OU**, Memorial University of Newfoundland
*Food chain length in a chemotactic bio-reactor model*

We talk about the effect and the impact of predator-prey interactions, diffusivity and chemotaxis on the ability of survival of multiple consumer levels in a predator-prey microbial food chain. We aim at answering the question of how many consumer levels can survive from a dynamical system point of view. To solve this issue on food-chain length, first we construct a chemotactic food chain model. A priori bounds of the steady state populations are obtained. Then under certain sufficient conditions combing the effect of conversion efficiency, diffusivity and chemotaxis parameters, we derive the co-survival of all consumer levels, thus obtaining the food chain length of our model. Numerical simulations not only confirm our theoretical results, but also demonstrate the impact of conversion efficiency, diffusivity and chemotaxis behavior on the survival and stability of various consumer levels.

**MARION WEEDERMANN**, Dominican University
*Coupled chemostat with nutrient chain and internal inhibition: a dynamic model for methanogenesis*

We propose a model describing two main stages of methanogenesis. The model includes two microorganisms, a nutrient chain and inhibition in case of abundance of one of the nutrients. One nutrient is supplied via the input to the feed vessel. As one species of microorganisms consumes this nutrient, a second nutrient is produced. While this second nutrient is essential for the growth of the second microorganism, a high concentration of this nutrient alters the conditions in the vessel so that the second microorganism cannot survive.

Using the inflow rate and the inflow concentration as parameters, we show that the system undergoes several bifurcations. The inhibition results in the possible bi-stability of two positive equilibria.

This work was motivated by installations of biogas reactors which experienced difficulties in reaching high production levels after start-up. We present numerical results which indicate that close observation and timely intervention are key to optimize the production of methane.

**GAIL WOLKOWICZ**, McMaster University, 1280 Main Street West, Hamilton, Ontario
*Competition in the presence of a virus in an aquatic system*

Recent research has determined that viruses are much more prevalent in aquatic environments than previously imagined. We derive a model of competition between two populations of bacteria for a single limiting nutrient in a chemostat where a virus is present. It is assumed that the virus can only infect one of the populations, the population that would be a more efficient consumer of the resource in a virus-free environment, in order to determine whether introduction of a virus can result in coexistence of the competing populations. Criteria for the global stability of the disease free and endemic steady states are obtained. It is also shown that it is possible to have multiple attracting endemic steady states, oscillatory behavior resulting from Hopf bifurcations, and a hysteresis effect. Mathematical tools that are used include Lyapunov functions, persistence theory, and bifurcation analysis.

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**YUAN YUAN**, Memorial University of Newfoundland
*Bogdanov–Takens Singularity in Van der Pol's Oscillator with Delayed Feedback*

In this talk, we study the classical Van der Pol's equation with delayed feedback. We investigate the Bogdanov–Takens singularity and bifurcation occurring in the system with the variation of the original parameters by using the normal form method, and predict the suitable choice in the feedback control in order to obtain the asymptotic stable state.

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**ZHIYONG ZHANG**, University of Alberta, 632 CAB, Math Department
*Asymptotic behavior of dissipative system with different constant end states*

In this talk we give some results on the global existence and the asymptotic behavior of solutions to the Cauchy problem for coupled initial data converging to constant states at infinity. The basic idea is to propose a linear convection-diffusion wave as the decay profile so as to approximate the limiting behavior as $x$ approaches infinity. The resulting nonlinear system then converges to the linear profile under certain smallness condition of the initial data and perturbation of the original system. We thus show that the evolution equations may be viewed as the composition of a linear diffusion wave and of a parabolic system which converges even faster.

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**XIAOQIANG ZHAO**, Memorial University of Newfoundland, St. John's, NL, A1C 5S7
*A Climate-Based Malaria Transmission Model with Age-structure*

In this talk, we present an age-structured model of malaria transmission. We first introduce the basic reproduction ratio for this model and then show that the disease-free periodic state is globally asymptotically stable when this ratio is less than one, and there exists at least one positive periodic state and the disease persists when it is greater than one. We further use these analytic results to study the malaria transmission cases in KwaZulu–Natal Province, South Africa, to determine how well they represent the biological system and, consequently, how useful their predictions are. Some sensitivity analysis of the basic reproduction ratio is performed, and in particular, the potential impact of climate change on seasonal transmission and populations at risk of the disease is analyzed.

This is joint work with Yijun Lou.

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**HUAIPING ZHU**, York University, Toronto
*Epidemic Models for Complex Networks with Demographics*

Usually the network models do not involve demographical dynamics. In this talk, I will introduce the generalized epidemic models for modeling infectious diseases spreading in complex networks. We will incorporate demographics and random mixing in to the modeling. The basic reproduction number will be calculated in terms of conditional probability for the correlated network models. The stationary state(s) will be calculated, we will also discuss their local and global asymptotic stability when they exist. Examples of application, including application to the modeling of HIV/AIDS will be presented.

This is a joint work with Professor Zhen Jin.

**XINGFU ZOU**, University of Western Ontario
*Impact of map dynamics on the dynamics of the associated delay reaction diffusion equation with Neumann condition*

We are concerned with the dynamics of a class of delay reaction diffusion equation with a parameter $\mu$. By letting $\mu \to +\infty$, such an equation is formerly reduced to an interval dynamical system. With the help of the famous Sarkovskii's theorem, we obtain some new yet simple sufficient conditions that assure the global stability of the delayed reaction diffusion equation with the parameter. We also give several examples to illustrate our main results.

This is a joint work with Taishan Yi.