Geometric Harmonic Analysis and Partial Differential Equations
Analyse harmonique géométrique et équations aux dérivées partielles

(Org: **Jie Xiao** (Memorial))

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A non-α-normal function whose derivative has finite area integral of order less than $2/\alpha$

Let $\mathcal{D}$ be the unit disk $\{z : |z| < 1\}$ in the complex plane. A function f, meromorphic in $\mathcal{D}$, is normal, denoted by $f \in N$, if $\sup_{z\in\mathcal{D}}(1-|z|^2)f^{\#}(z) < \infty$, where $f^{\#}(z) = |f'(z)|/\big(1+|f(z)|^2\big)$. For $\alpha > 1$, a meromorphic function f is called α-normal if $\sup_{z\in\mathcal{D}}(1-|z|^2)^{\alpha}f^{\#}(z) < \infty$. H. Allen and C. Belna [1] have proved that there is an analytic function f_1, defined in $\mathcal{D}$, such that

$$\iint_{\mathcal{D}} |f_1'(z)|\,dx\,dy < \infty$$

but $f_1 \notin N$. S. Yamashita [3] sharpened this result by showing that for another analytic function f_2 which does not belong to N it holds

$$\iint_{\mathcal{D}} |f_2'(z)|^p\,dx\,dy < \infty \tag{1}$$

for all p, $0 < p < 2$. Further, H. Wulan [2] studied more the function f_2 and showed that $f_2 \notin \bigcup_{0<p<\infty} Q_p^{\#}$ but $f_2 \in \bigcap_{0<p<\infty} M_p^{\#}$. We construct a class of analytic functions f_s which satisfy (1) for $0 < p < \frac{2}{\alpha}$ but $f_s \notin N^{\alpha}$ for $\alpha > 1$. Further, the question if f_s belongs or not to $\bigcup_{0<p<\infty} M_p^{\#}$ is considered.

References

[1] H. Allen and C. Belna, *Non-normal functions $f(z)$ with $\iint_{|z|<1}|f'(z)|\,dx\,dy < \infty$*. J. Math. Soc. Japan **24**(1972), 128–132.

[2] H. Wulan, *A non-normal function related Q_p spaces and its applications.* In: Progress in Analysis **I, II**, World Sci. Publ., River Edge, NJ, 2003, 229–234.

[3] S. Yamashita, *A non-normal function whose derivative has finite area integral of order $0 < p < 2$.* Ann. Acad. Sci. Fenn. Ser. Math. **4**(1978/1979), 293–298.

JINGYI CHEN, UBC

Lagrangian Mean Curvature flow for entire Lipschitz graphs

We prove existence of longtime smooth solutions to mean curvature flow of entire Lipschitz Lagrangian graphs. As an application of the estimates for the solution, we establish a Bernstein type result for translating solitons.

The results are from joint work with Albert Chau and Weiyong He.

MIROSLAV ENGLIS, Mathematics Institute, Academy of Sciences, Czech Republic

Hankel operators and the Dixmier trace

We discuss the membership of (big) Hankel operators H_f on weighted Bergman spaces in the Dixmier class. For the unit disc and f holomorphic, $H_{\overline{f}}$ is in this class if and only if f' belongs to the Hardy one-space H^1. On the unit ball in $\mathbf{C}^n$, $n \geq 2$, there is an analogous result for any f smooth on the closed ball (not necessarily conjugate-holomorphic), which is reminiscent of the trace formula of Helton and Howe. The last result extends also to arbitrary smoothly bounded strictly pseudoconvex domains in $\mathbf{C}^n$, where the formula for the Dixmier trace turns out to involve the Levi form, thus exhibiting an interesting link with the geometry of the domain. The proofs involve analysis of pseudodifferential operators on the boundary of the domains.

The case of the disc is joint work with Richard Rochberg, while the case of the ball and of pseudoconvex domains are joint with Kunyu Guo and Genkai Zhang.

DAVID HARTENSTINE, Western Washington University, Bellingham, WA, USA
Dual Brunn–Minkowski Inequality for $(n-1)$-Capacity

A dual capacity Brunn–Minkowski inequality is established for the $(n-1)$-capacity of radial sums of star bodies in $\mathbb{R}^n$. This inequality is a counterpart for the p-capacity of Minkowski sums of convex bodies in $\mathbb{R}^n$ for $1 \leq p < n$, proved by Borell, Colesanti and Salani. When $n \geq 3$, the dual inequality follows from an inequality of Bandle and Marcus, but our new proof allows us to establish an equality condition. In the $n = 2$ case, we use different techniques to establish the inequality and a different equality condition. These results show that in a sense $(n-1)$-capacity has the same status of volume in that it plays the role of its own dual set function in the Brunn–Minkowski and dual Brunn–Minkowski theories.

This is joint work with Richard Gardner.

CHIN-CHENG LIN, National Central University, Chung-Li 320, Taiwan
Hardy spaces associated with Schrödinger operators on the Heisenberg group

Let $L = -\Delta_{\mathbb{H}^n} + V$ be a Schrödinger operator on the Heisenberg group $\mathbb{H}^n$, where $\Delta_{\mathbb{H}^n}$ is the sub-Laplacian and the nonnegative potential V belongs to the reverse Hölder class $B_{\frac{Q}{2}}$ and Q is the homogeneous dimension of $\mathbb{H}^n$. The Riesz transforms associated with the Schrödinger operator L are bounded from $L^1(\mathbb{H}^n)$ to $L^{1,\infty}(\mathbb{H}^n)$. The L^1 integrability of the Riesz transforms associated with L characterizes a certain Hardy type space denoted by $H^1_L(\mathbb{H}^n)$ which is larger than the usual Hardy space $H^1(\mathbb{H}^n)$. We define $H^1_L(\mathbb{H}^n)$ in terms of the maximal function with respect to the semigroup $\{e^{-sL} : s > 0\}$, and give the atomic decomposition of $H^1_L(\mathbb{H}^n)$. As an application of the atomic decomposition theorem, we prove that $H^1_L(\mathbb{H}^n)$ can be characterized by the Riesz transforms associated with L.

PAOLO SALANI, Università degli Studi di Firenze, Italy
An overdetermined problem for a Finsler–Laplacian

I will present some results of a recent joint paper with A. Cianchi. The purpose is to embed the famous Serrin's symmetry result (Arch. Rational Mech. Anal., 1971) in a general symmetry principle for solutions to overdetermined elliptic problems, where the relevant symmetry is not necessarily of spherical type. The underlying idea of our contribution is that a symmetry result holds for any overdetermined problem involving an elliptic operator (with quadratic growth) from a suitable class, provided that the additional boundary condition imposed on the gradient of the solution matches the structure of the differential operator. The resulting symmetry of the domain (and of the corresponding solution u) reflects, in turn, the symmetry of the operator, that is: a solution exists if and only if the domain Ω is a ball in an appropriate Finsler metric associated with the operator (moreover the level sets of u are homothetic to Ω).

XINWEI YU, 527 CAB, University of Alberta
On the Conserved Quantities of the 2D Surface Quasi-geostrophic Equation

The 2D surface quasi-geostrophic (SQG) equation can be seen as a 2D model equation for the 3D incompressible Euler equations. In this talk we will discuss necessary and sufficient conditions, characterized by Besov and Triebel–Lizorkin type spaces, for the conservation of various conserved quantities of the 2D SQG equation. These conditions can help in the searching of rigorous mathematical framework modeling turbulent flows.

DENGLIN ZHOU, University of Waterloo, 200 Univ. Ave. W., Waterloo, ON, N2L 3G1
Gaps in the ratios of the spectrum of Laplacians on fractals

In contrast to the classical situation, it is known that many Laplacian operators on fractals have gaps in their spectra. This surprising fact means there can be no limit in the Weyl counting formula and it is a key ingredient in proving that the convergence of Fourier series on fractals can be better than in the classical setting. Recently, it was observed that the Laplacian on the Sierpinski gasket has the stronger property that there exist intervals which contain no ratios of eigenvalues. In this talk, we give general criteria for this phenomenon and show that Laplacians on many interesting classes of fractals satisfy our criteria.

This is a joint work with Kathryn Hare.