A simple discretization scheme for nonnegative diffusion processes, with applications in option pricing

The Cox–Ingersoll–Ross (CIR) process is broadly used in financial engineering, mostly since it assumes nonnegative values. Applying a standard Euler discretization scheme for simulations is problematic, as nothing prevents the simulated values to be negative. Several fixes are found in the literature, most involving the normal distribution. We propose a scheme producing nonnegative discrete approximations for nonnegative diffusions (e.g. CIR), in which nonnegativity is secured by sampling from a nonnegative distribution. We use the martingale problem to establish convergence of the approximating processes to the target diffusion. The proposed scheme is suitable for several applications in pricing derivative securities.