Let $k$ be a field and let $G$ be a finite group. We study ungraded, commutative $k$-algebras $R$ on which $G$ acts by $k$-algebra automorphisms rendering $R$ a projective $kG$-module. Such projective $k - G$-algebras and their invariants have a beautiful structure theory and they arise in invariant theory in the study of certain localisations.

In the case of $p$-groups in characteristic $p$, we describe the algebra $D_k$, which is a generator in the category of commutative, projective $k - P$-algebras, and we give explicit generators and relations for the invariant ring $D_k^P$. We also define and describe simple cyclic projective $k - P$-algebras, which include the Galois extensions of $k$, and universal projective $k - P$-algebras, from which all the others can be constructed by forming quotients and “extending invariants”.

This is joint work with my colleague C. F. Woodcock (Kent).