Patching techniques originally used in inverse Galois theory are based on factorization theorems for invertible matrices. This talk generalizes these factorization results from $\text{Gl}_n$ to rational linear algebraic groups. As a consequence, one obtains a local global principle for homogeneous spaces under such groups. One application is a new proof of the recent result of Parimala and Suresh on the maximal dimension of anisotropic quadratic forms over $p$-adic function fields ($u$-invariant). The same approach yields results on the period-index problem for central simple algebras.