Additive group actions associated to derivations of \( R[X, Y, Z] \) with a slice

This talk features a simple family of locally nilpotent \( R \)-derivations of \( R[X, Y, Z] \) with a slice, where \( R = \mathbb{C}[a, b] \). Equivalently, this is a family of \( \mathbb{G}_a \)-actions on \( \mathbb{A}^5 \) such that \( \mathbb{A}^5 = V \times \mathbb{A} \), where \( V \) is the variety defined by the algebra of invariants, and \( \mathbb{G}_a \) acts by translation. We show that \( V \) is an \( \mathbb{A}^2 \)-fibration over \( \mathbb{A}^2 \), but it is unknown whether this is a trivial fibration. Note that \( V \) has the form \( \text{Spec}(B/sB) \), where \( B = R[X, Y, Z] \) and \( s \in B \) is the corresponding slice. We give a method for finding \( f \in B \) of degree smaller than \( s \) such that \( B/fB \) and \( B/sB \) are isomorphic as fibrations. However, it is not known whether \( f \) is a slice for any locally nilpotent derivation of \( B \). These examples are motivated by the Vénéreau polynomials \( v \in L = \mathbb{C}[x, y, z, u] \). It was shown by the author that, if \( K = \mathbb{C}[x, v] \), then \( L[t] = K[X, Y, Z] \). The main idea is to study \( d/dt \) as a \( K \)-derivation of \( K[X, Y, Z] \).