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Collision of two solitons for nonintegrable gKdV equations

In this talk, I will report on recent results in collaboration with Frank Merle concerning the collision of two solitons for the subcritical gKdV equations $\partial_t u + \partial_x\big(\partial_x^2 u + g(u)\big) = 0$ on $\mathbf{R}$. Under general assumptions on $g(u)$, there exist stable solutions of the form $u(t,x) = Q_c(x-ct)$, called *solitons*.

Considering $0 < c_2 < c_1$, there exists a unique solution $u(t)$ such that $\lim_{t\to-\infty} \|u(t) - Q_{c_1}(.-c_1 t) - Q_{c_2}(.-c_2 t)\|_{H^1(\mathbf{R})} = 0$. Formally, the solitons have to collide. We study this collision, i.e., the behavior of the solution $u(t)$ for t bounded and for $t \to +\infty$, in the case where c_2 is small.

The first result concerns the global stability of the two soliton structure. Indeed, we prove that the two solitons survive the collision, with a possible residue and a possible change of sizes (and speeds) small with respect to Q_{c_2}. Moreover, we show monotonicity properties: the size of the large soliton does not decrease and the size of the small soliton does not increase through the collision.

Second, we focus on the quartic case $g(u) = u^4$, for which we give a sharp description of the collision. We prove that the residue is small but not zero. The collision is inelastic but very close to be elastic. This is in contrast with the integrable cases ($g(u) = u^2$ or u^3) for which the collision is elastic (zero residue).

Finally, we exhibit special symmetric solutions which extend to the nonintegrable case the notion of 2-soliton for KdV.