WALTER CRAIG, McMaster University, Department of Mathematics & Statistics
Navier–Stokes equations and their Kolmogorov spectra

Suppose that $u(x,t)$ is a (possibly weak) solution of the Navier–Stokes equations on all of $\mathbb{R}^3$, or on the torus $\mathbb{R}^3/\mathbb{Z}^3$. Let $\hat{u}(k,t)$ denote its Fourier transform. Then the *energy spectrum* of $u(\cdot,t)$ is the spherical integral

$$E(\kappa,t) = \int_{|k|=\kappa} |\hat{u}(k,t)|^2 \, dS(k), \quad 0 \le \kappa < \infty,$$

or alternatively, a suitable approximate sum. An argument invoking scale invariance and dimensional analysis given by Kolmogorov in 1941, and subsequently refined by Obukov, predicts that large Reynolds number solutions of the Navier–Stokes equations in three dimensions should obey

$$E(\kappa,t) \sim C\kappa^{-5/3},$$

at least in an average sense and over some of the range of κ. I will describe a global estimate on weak solutions in the norm $|\mathcal{F}\partial_x u(\cdot,t)|_\infty$ which gives bounds on a solution's ability to satisfy the Kolmogorov law. The result gives rigorous upper and lower bounds on the inertial range, and an upper bound on the time of validity of the Kolmogorov spectral regime.

This is joint work with Andrei Biryuk.