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Flow control by small forces

Consider the motion of the ideal incompressible fluid on the surface of 2-d torus $\mathbf{T}^2$ described by the Euler equations

$$\frac{\partial u}{\partial t} + (u, \nabla)u + \nabla p = 0, \quad \nabla \cdot p = 0. \tag{1}$$

Let u_0, u_1 be two stationary (time independent) solutions of (1). Consider the nonhomogeneous Euler equations

$$\frac{\partial u}{\partial t} + (u, \nabla)u + \nabla p = f, \quad \nabla \cdot p = 0, \tag{2}$$

where $f = f(x,t)$ is some external force. We say that the force f transfers u_0 into u_1 during the time T, if the solution of (2) satisfying $u(x,0) = u_0(x)$, satisfies also $u(x,T) = u_1(x)$.

Theorem *For any stationary solutions u_0, u_1 having equal energies and momenta, and for any $\varepsilon > 0$ there exist $T > 0$ and a force $f \in C^\infty(\mathbf{T} \times [0,T])$ such that f transfers u_0 into u_1 during the time T, and*

$$\max_{0 \le t \le T} \|f\|_{L^2} + \int_0^T \|f(\cdot,t)\|_{L^2}\, dt < \varepsilon. \tag{3}$$

So, the flow may be controlled by a small (in L^2) external force; most of the job is done by the fluid itself. One consequence of this fact is the absence of integrals of the Euler equations which are continuous in L^2 and different from the energy and momentum.

The proof is done by construction and uses the multiphase flow approximation for complex flows.