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C-minimal structures

A C -relation is a ternary relation first-order interpreting a tree which is a meet-semi-lattice, the domain of the C -relation being then a covering set of branches with no isolated branch. A set M , equipped with a C -relation and possibly an additional structure, is called C -minimal if any definable subset of M is definable without quantifiers in the pure language of C , and if the same holds in any elementarily equivalent structure.

As an example, a C -relation defined on a field and compatible with the two operations derives from a valuation, in the sense that $C(x, y, z)$ means $v(x - y) < v(y - z)$, and it is C -minimal iff the field is algebraically closed. But a C -relation defined on, and compatible with, a group need not derive from a valuation.

A C -minimal structure is algebraically bounded in the sense that finite uniformly definable sets have a bounded size. On the other hand the algebraic closure need not satisfy the exchange principle. C -minimal structures with exchange are “geometric” in the sense of Zilber. In this context, we may ask the question of the trichotomy: Is it possible to define a group in nontrivial structures? To define a field in nonlocally modular structures? To classify some of these structures?

The aim is to extend once more the range of applications of the powerful machinery of the stability. O -minimality allowed handling some ordered structures and the hope is that C -minimality will do the same with geometric C -minimal structures.