Kinetic Methods in Partial Differential Equations
Méthodes cinétiques en EDP
(Org: **François Castella** (Rennes) and/et **Reinhard Illner** (Victoria))

NAOUFEL BEN ABDALLAH, Université de Toulouse, France
High field limit for the Boltzmann equation

We consider the High field limit of the Boltzmann equation of semiconductors

$$\partial_t f_\varepsilon + v \cdot \nabla_x f_\varepsilon + \frac{1}{\varepsilon}\big(E \cdot \nabla_v f_\varepsilon - Q(f_\varepsilon)\big) = 0.$$

The collision operator Q is nonlinear and the parameter ε which measures the mean free path as well as the inverse scale of the force field E is assumed to be small.

The limiting equation is a nonlinear conservation law. Thanks to a Hilbert expansion method and to the L^1 contractivity of both the kinetic and the limiting equations, we show the convergence of the kinetic solutions towards the solution of the limiting conservation whenever the latter is smooth.

Then, we derive a series of entropies for the kinetic equation, which "converge" to the convex entropies of the limiting conservation law. By using these entropies, we prove the convergence as ε tends to zero, of the kinetic solutions f_ε towards the unique entropy solution, even when shocks do appear. The same entropy allows to construct kinetic profiles for entropic shocks. Diffusive correction can be handled by a linearized version of these entropies.

This is a work in collaboration with Hédia Chaker (Tunis) and Christian Schmeiser (Vienna).

MARTIAL AGUEH, University of Victoria
Asymptotic behavior for p-Laplacian type equations

The long-time asymptotics for the doubly nonlinear diffusion equations $\rho_t = \text{div}(|\nabla\rho^m|^{p-2}\nabla\rho^m)$ in R^n, is studied for $p > 1$ and $m > \frac{n-p}{n(p-1)}$. The non-negative solutions of the equation are shown to behave asymptotically, as $t \to \infty$, like Barenblatt-type solutions, and a polynomial decay is established for the convergence with respect to the L^1-norm. The rate of decay is proved to be optimal when $m \geq \frac{n-p+1}{n(p-1)}$. The method used is based on optimal transportation arguments when $m \geq \frac{n-p+1}{n(p-1)}$, and on a linear analysis when $\frac{n-p}{n(p-1)} < m < \frac{n-p+1}{n(p-1)}$.

GUILLAUME BAL, Columbia

NASSIF GHOUSSOUB, University of British Columbia
Bessel pairs and optimal Hardy and Hardy–Rellich inequalities

We give necessary and sufficient conditions on a pair of positive radial functions V and W on a ball Ω of radius R in $\mathbf{R}^n$, $n \geq 2$, so that the following inequalities hold for all $u \in C_0^\infty(\Omega)$:

$$\int_\Omega V(x)|\nabla u|^2\,dx \geq \int_\Omega W(x)u^2\,dx \tag{1}$$

and

$$\int_B V(x)|\Delta u|^2\,dx \geq \int_B W(x)|\nabla u|^2\,dx + (n-1)\int_B \Big(\frac{V(x)}{|x|^2} - \frac{v'(|x|)}{|x|}\Big)|\nabla u|^2\,dx. \tag{2}$$

We then identify a large number of such couples (V, W)—that we call Bessel pairs—and the best constants in the corresponding inequalities. This will allow us to complete, improve, extend, and unify most related results—old and new—about Hardy and Hardy–Rellich type inequalities which were obtained by Caffarelli–Kohn–Nirenberg, Brezis–Vázquez, Wang–Willem, Adimurthi–Chaudhuri–Ramaswamy, Filippas–Tertikas, Adimurthi–Grossi–Santra, as well as some very recent work by Tertikas–Zographopoulos, Liskevich–Lyachova–Moroz, and Blanchet–Bonforte–Dolbeault–Grillo–Vasquez, among others.

This is joint work with Amir Moradifam.

THIERRY GOUDON, Project Team SIMPAF, INRIA Lille Nord Europe

Particles subject to a high oscillating force field

We are interested in asymptotic problems for kinetic equations where the small parameter is intended to describe both the (high) strength and the (fast) period of oscillations of the force field the particles are subject to.

In this problem the force field is derived from random principles, which in turn imply some dissipation mechanism and lead to effective equations involving diffusion with respect to the velocity variable.

FREDERIC HERAU, Université de Reims

Exponential hypocoercivity for simple kinetic models with boundary conditions

We present in this talk some recent results about the application of L^2 hypocoercive methods to the explicit exponential trend to the equilibrium for some simple kinetic models (Fokker–Planck equation and linear relaxation) in bounded domains.

REINHARD ILLNER, University of Victoria, Department of Mathematics and Statistics, PO Box 3060 STN CSC, Victoria, BC W8W 3R4

On Stop-and-Go Waves in Dense Traffic

Starting from a Vlasov-type kinetic model we derive nonlocal macroscopic models generalizing the models of conservation type first introduced by Aw–Rascle and Zhang. We discuss reasonable examples of braking and acceleration forces as functions of density and relative speed. Removing the nonlocality by Taylor expansions produces nonlinear PDEs for both braking and acceleration scenarios. A traveling wave ansatz produces stop-and-go waves in good qualitative agreement with practical observations.

PIERRE-EMMANUEL JABIN, University of Nice, Parc Valrose, 06108 Nice cedex 2, France

Mean field limits for interacting particles

The behaviour of a large number of particles interacting with a singular potential is studied. In the repulsive case and for almost all initial configurations, it is possible to show a stability result for the flow, quantitative and uniform in the number of particles. In particular, the dynamics remains close to any regularized flow, and therefore to the dynamics predicted by the Vlasov equation at the limit.

BOUALEM KHOUIDER, University of Victoria

An inviscid regularization for the SQG equations

One outstanding open question of fundamental interest in fluid mechanics concerns the existence of long-time smooth solutions for the 3D Euler equations. It is argued in the literature that the surface quasi-geostrophic (SQG) equations bear some resemblance with the 3D Euler vorticity equations and as such they provide an interesting model for studying long time behaviour of solutions in a context similar to Euler equations. As an alternative to previous interesting approaches by others, we propose here an inviscid regularization for the SQG equations to facilitate the analysis. The new regularization yields a necessary and sufficient condition, satisfied by the regularized solution, when a regularization parameter tends to zero, for the solution of the original SQG equations to develop a singularity in finite time. As opposed to the commonly used viscous regularization, the inviscid equations derived here conserve a modified energy.

Therefore, the new regularization provides an attractive numerical procedure for finite time blow up testing. In particular, we prove that, if the initial condition is smooth, then the regularized solution remains as smooth as the initial data for all times. Some numerical tests will be presented as well.

Joint work with Edriss Titi.

ANTOINE MELLET, University of British Columbia, Vancouver, BC
Analysis of a fluid-kinetic system of equations

We study a coupled system of kinetic and fluid equations modeling fluid-particles interactions arising in sprays, aerosols or sedimentation problems. More precisely, we consider a Vlasov–Fokker–Planck equation coupled to compressible Navier–Stokes equation via a drag force. We establish the existence of solutions and we rigorously derive the asymptotic regime corresponding to a strong drag force and a strong Brownian motion.

PAUL MILEWSKI, University of Wisconsin, Madison
A simple model for swarming with nonlocal sensing

We present a simple conservation law model describing the population level swarming of organisms. The organism's speed of motion results from the combination of a nonlocal, long range, attraction and short range repulsion. The kernel of the long range attraction term is not symmetric and can be thought of as modeling the response of higher organisms to visual cues where the organisms sense others primarily in the direction in which they are moving. The resulting model has compact travelling waves (swarms) whose speed increases with the swarm "mass" up to a largest swarm. There are also front and periodic solutions. We discuss a possible kinetic model that leads to our conservation law and some extensions of our macroscopic model.

This is joint work with Xu Yang.

MARJOLAINE PUEL, Toulouse

PIERRE RAPHAEL, Université Paul Sabatier, 31062 Toulouse, France
On the stability of ground states and the singularity formation for the gravitational Vlasov–Poisson system

I will consider the three dimensional relativistic gravitational Vlasov–Poisson system

$$\begin{cases}\partial_t f + \frac{v}{\sqrt{1+|v|^2/c^2}} \cdot \nabla_x f - \nabla \phi_f \cdot \nabla_v f = 0, \\ f(t=0,x,v) = f_0(x,v) \geq 0, (t,x,v) \in R_+ \times R^3 \times R^3 \end{cases} \tag{1}$$

where ϕ_f is the Poisson gravitational field:

$$\phi_f(x) = -\frac{1}{4\pi}\frac{1}{|x|} \star \rho_f, \quad \rho_f(x) = \int_{R^3} f(x,v)\, dv, \tag{2}$$

and $c \in]0, +\infty]$ is the 'light speed'. The value $c = +\infty$ recovers the classical Vlasov–Poisson system which is a nonlinear transport equation describing the mechanical state of a stellar system subject to its own gravity. A well known fact is that smooth solutions to the classical system are global in time while the relativistic system $c < +\infty$ may develop finite time blow up singularities. I will first discuss both in the classical and relativistic settings the question of the existence and stability of ground states stationary solutions for these systems. I will then focus onto the singularity formation problem in the relativistic case and prove the existence of a *stable* self similar blow up dynamics corresponding to a concentration phenomenon for the distribution function.

This is a joint program with Mohammed Lemou (IMT, Toulouse) and Florian Mehats (IRMAR, Rennes).

CATHERINE SULEM, University of Toronto, Department of Mathematics
Water waves over a random topography

We discuss the problem of nonlinear wave motion of the free surface of a body of fluid over a variable bottom. The object is to describe the character of wave propagation in a long wave asymptotic regime. We assume that the bottom of the fluid region can be described by a stationary random process whose variations take place on short length scales. It is a problem in homogenization theory. Our principal result is the derivation of effective equations and a consistency analysis.