

---

**Representations of Finite and Algebraic Groups**  
**Représentations des groupes finis et des groupes algébriques**  
(Org: **Gerald Cliff** (Alberta) and/et **Anna Stokke** (Winnipeg))

---

---

**PETER CAMPBELL**, University of Bristol, Bristol, U.K.

*K-types for principal series representations of  $GL(3)$*

Let  $\mathfrak{o}$  be the ring of integers of a  $p$ -adic field  $F$ , then  $K = GL(3, \mathfrak{o})$  is a maximal compact subgroup of the  $p$ -adic group  $G = GL(3, F)$ . On restriction to  $K$ , a principal series representation of  $G$  necessarily decomposes as the direct sum of smooth irreducible representations of  $K$  each appearing with finite multiplicity. We will give a description of this decomposition, with particular emphasis on the unramified case, and outline an application to an analogue of the Steinberg representation for the finite group  $GL(n, \mathfrak{o}/\mathfrak{p}^\ell)$  where  $\mathfrak{p}$  is the prime ideal of  $\mathfrak{o}$ .

This is joint work with Monica Nevins (University of Ottawa).

---

**XUEQING CHEN**, University of Wisconsin–Whitewater

*Quivers, Ringel-Hall Algebras and Lie Theory*

Let  $\mathfrak{g} = \mathfrak{g}(C)$  be the Kac–Moody Lie algebra associated to a Cartan matrix  $C$  and  $\mathbf{U} = \mathbf{U}_v(\mathfrak{g})$  its quantum group. A key feature in quantum groups is the presence of several natural bases (like the PBW-basis and the canonical basis). There are different approaches to the construction of the canonical basis: algebraic approach, geometric approach and Ringel–Hall algebra approach. In this talk, we start by recalling the basic theory of quivers and Ringel–Hall algebras, paying special attention to Gabriel’s Theorem and Ringel–Green’s work on the realization of quantum groups and Lie algebras by using Hall algebras of finite dimensional associative algebras. We will then recall algebraic and Ringel–Hall algebra approaches to a PBW basis and a canonical basis of  $\mathbf{U}$  when  $C$  is of finite or affine type. Meanwhile, the root vectors in Ringel–Hall algebras will be discussed. Finally, we shall go on to discuss some of the many further developments and applications of the theory.

---

**JAYDEEP CHIPALKATTI**, University of Manitoba

*The higher transvectants are redundant*

Transvectants were discovered in the nineteenth century by the German school of classical invariant theorists. In modern terminology, they encode the decomposition of the tensor product of two  $SL_2$ -representations. Given binary forms  $A$  and  $B$  of orders  $d, e$  respectively, their  $r$ -th transvectant  $T_r = (A, B)_r$  is a form of order  $d + e - 2r$  whose coefficients are bilinear in those of  $A, B$ . We classify all quadratic syzygies in the  $T_r$ , and show that as a consequence, the higher transvectants  $\{T_r\}_{r \geq 2}$  can be entirely recovered from  $T_0, T_1$ .

This is joint work with A. Abdesselam from Université Paris XIII.

---

**ALLEN HERMAN**, University of Regina, Regina, SK, S4S 0A2

*The Group of Ring Automorphisms of a Rational Group Algebra*

Ring automorphisms of the rational group algebra  $\mathbb{Q}G$  of a finite group  $G$  come in two types: inner automorphisms that leave every simple component of  $\mathbb{Q}G$  invariant, and outer automorphisms that interchange at least two of these simple components.

In order to detect the existence of outer automorphisms of  $\mathbb{Q}G$ , one must be able to determine whether or not two simple components of  $\mathbb{Q}G$  are ring isomorphic. It is automatic that two isomorphic simple components will have isomorphic centers,

equal dimension, and the same local Schur indices at each rational prime. However, an example of a group will be given to show that these conditions are not sufficient. It has two simple components that are ring isomorphic but not Morita equivalent. Procedures for determining whether two simple components of  $\mathbb{Q}G$  are ring isomorphic will be discussed.

---

**GREG LEE**, Lakehead University, Thunder Bay, Ontario

*Nilpotent symmetric units in group rings*

The group ring  $KG$  of a group  $G$  over a field  $K$  has a natural involution  $*$  sending each group element to its inverse. The elements fixed by this involution are said to be symmetric.

We will examine the symmetric units in  $KG$ , and discuss the conditions under which they satisfy a group identity. In particular, we will explore the conditions under which the symmetric units are nilpotent. Until recently, all of the known results concerned torsion groups. However, new results allow us to consider groups with elements of infinite order.

---

**DAVID MCNEILLY**, Alberta

---

**FERNANDO SZECHTMAN**, University of Regina

*Modular Reduction of the Steinberg Lattice of the General Linear Group*

The mod  $\ell$  reduction of the Steinberg lattice of  $G = \mathrm{GL}(n, q)$  is known not to be irreducible when  $\ell$  divides the index  $[G : B]$  of the upper triangular group  $B$  in  $G$ . In 2003 R. Gow produced a canonical filtration for this reduced module, and conjectured that all non-zero factors are irreducible. The bottom factor is the socle, which has been known to be irreducible for some time. We consider the next factor of Gow's filtration, lying just above the socle, and prove it to be irreducible in the particular case when  $\ell$  divides  $q + 1$ .

---

**QIANGLONG WEN**, University of Alberta, 114 St – 89 Ave, Edmonton, Alberta, Canada, T6G 2E1

*Some Irreducible Characters of  $\mathrm{GL}(2, \mathbb{Z}/p^n\mathbb{Z})$  and  $\mathrm{GL}(3, \mathbb{Z}/p^n\mathbb{Z})$*

Nowadays there is considerable interest in the representations of  $\mathrm{GL}(n, \mathbb{Z}_p)$ , where  $\mathbb{Z}_p$  are the  $p$ -adic integers. Since every continuous irreducible representation of  $\mathrm{GL}(n, \mathbb{Z}_p)$  comes from a representation of  $\mathrm{GL}(n, \mathbb{Z}_p/p^m\mathbb{Z}_p)$  and  $\mathbb{Z}_p/p^m\mathbb{Z}_p \cong \mathbb{Z}/p^m\mathbb{Z}$ , I focus on finding some irreducible characters of  $\mathrm{GL}(n, \mathbb{Z}/p^m\mathbb{Z})$ . Clifford Theory gives us a method to construct irreducible characters of a group  $G$ , by inducing up certain irreducible characters of subgroups  $H$  of  $G$ . I apply Clifford Theory to construct three types of irreducible characters of groups  $\mathrm{GL}(2, \mathbb{Z}/p^n\mathbb{Z})$  and  $\mathrm{GL}(3, \mathbb{Z}/p^n\mathbb{Z})$ .