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An inequality on \( m \)-term approximation by ultraspherical polynomials and its applications

In this talk, I shall show a useful inequality on \( m \)-term approximation by ultraspherical polynomials on \([-1, 1]\). As an application, I shall show how to use this inequality to construct a sequence of polynomials \( \psi_j, j = 1, 2, \ldots \) with the following properties:

(i) \( \psi_j \in \text{span}\{P_{2^{j-1}+1}^\lambda, P_{2^{j-1}+2}^\lambda, \ldots, P_{2^j}^\lambda\} \), where \( P_k^\lambda \) denotes the usual ultraspherical polynomial of degree \( k \) and index \( \lambda \) on \([-1, 1]\).

(ii) \( \|\psi_j\|_{2,\lambda} \approx \|\psi_j\|_{\infty} \) with the constant of equivalence independent of \( j \), here \( \|\cdot\|_{2,\lambda} \) denotes the \( L^2 \) norm on \([-1, 1]\) computed with respect to the weight \((1-t^2)^\lambda\).