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Integral representations of the derivatives of functions in \( \mathcal{H}(b) \)

Let \( H^p(\mathbb{C}_+) \) stand for the Hardy space of the upper half plane \( \mathbb{C}_+ \), and for \( \varphi \in L^\infty(\mathbb{R}) \), let \( T_\varphi \) stand for the Toeplitz operator defined on \( H^2(\mathbb{C}_+) \) by

\[
T_\varphi(f) := P_+(\varphi f), \quad (f \in H^2(\mathbb{C}_+)),
\]

where \( P_+ \) denotes the orthogonal projection of \( L^2(\mathbb{R}) \) onto \( H^2(\mathbb{C}_+) \). Then, for \( \varphi \in L^\infty(\mathbb{R}), \|\varphi\|_\infty \leq 1 \), the de Branges–Rovnyak space \( \mathcal{H}(\varphi) \), associated to \( \varphi \), consists of those \( H^2(\mathbb{C}_+) \) functions which are in the range of the operator \( (\text{Id} - T_\varphi T_\varphi^{1/2}) \).

It is a Hilbert space when equipped with the inner product

\[
\langle (\text{Id} - T_\varphi T_\varphi^{1/2}) f, (\text{Id} - T_\varphi T_\varphi^{1/2}) g \rangle_\varphi = \langle f, g \rangle_2,
\]

where \( f, g \in H^2(\mathbb{C}_+) \ominus \ker(\text{Id} - T_\varphi T_\varphi^{1/2}) \). In particular, if \( b \) is an inner function, then \( (\text{Id} - T_b T_b^{1/2}) \) is an orthogonal projection and \( \mathcal{H}(b) \) is a closed (ordinary) subspace of \( H^2(\mathbb{C}_+) \) which coincides with the so-called model spaces \( K_b = H^2(\mathbb{C}_+) \ominus bH^2(\mathbb{C}_+) \).

We give some integral representations for the boundary values of derivatives of functions of the de Branges–Rovnyak spaces \( \mathcal{H}(b) \), where \( b \) is an extreme point of the unit ball of \( H^\infty(\mathbb{C}_+) \).