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Jordan Arens irregularity

Given a Banach algebra $\mathcal{A}$, its product admits two canonical extensions to the bidual, yielding the left and right Arens products. If these coincide, $\mathcal{A}$ is called Arens regular; this is the case, e.g. for all C^*-algebras $\mathcal{A}$. However, most Banach algebras arising in abstract harmonic analysis are preduals of Hopf–von Neumann algebras, and multiplication in the second dual is typically highly irregular. The degree of irregularity can be measured through the so-called left and right topological centres, i.e., the sets of elements in $\mathcal{A}^{**}$ for which the left, resp. right, Arens product is separately w^*-w^*-continuous. Arens regularity means precisely that both topological centres coincide with $\mathcal{A}^{**}$; if the left, resp. right, topological centre equals $\mathcal{A}$, following Dales–Lau, we call $\mathcal{A}$ left, resp. right, strongly Arens irregular (SAI). There are natural examples of Banach algebras (due to Dales–Lau and myself) which are left but not right SAI—such as the algebra $\big(\mathcal{T}(L_2(\mathcal{G})),*\big)$, for non-compact $\mathcal{G}$, equipped with a certain convolution type product I introduced and studied; its right topological centre can also be described explicitly for all discrete groups $\mathcal{G}$.

A canonical way to symmetrize a given Banach algebra is to consider the corresponding Jordan product. This prompts the question to what extent this symmetry is carried over to the second dual—in other words: what is the (algebraic) centre of the second dual of a Jordan algebra? We are thus led to the notion of Jordan Arens (ir)regularity.

We shall prove that for any discrete ICC group $\mathcal{G}$, the centre of the bidual of $\ell_1(\mathcal{G})$ endowed with the Jordan product, is exactly $\ell_1(\mathcal{G})$. The proof relies on our simultaneous left/right factorization theorem for bounded sequences in $\ell_\infty(\mathcal{G})$ through elements in $\ell_1(\mathcal{G})^{**}$ and a single function in $\ell_\infty(\mathcal{G})$. As a consequence, we shall derive that for the same class of groups, $\big(\mathcal{T}(\ell_2(\mathcal{G})),*\big)$ is Jordan SAI as well.

This is joint work with my Master's student, Chris Auger.