Some new results will be presented on the study of the second part of Hilbert’s 16th problem. After a brief review of the problem, the main attention of this talk is focused on the weakened Hilbert’s 16th problem for higher-order Hamiltonian systems. First, a summary will be given for the results of odd order systems: \( H(5) \geq 5^2 - 1 \), \( H(7) \geq 7^2 \), \( H(9) \geq 9^2 - 1 \), and \( H(11) \geq 11^2 \), then the particular attention is given to rarely-considered even order systems. With the aid of the detection function method and normal form theory, both global and local bifurcation analyses are employed to show that a quintic Hamiltonian system under a 6th-order perturbation can generate at least 35 limit cycles, \( i.e., H(6) \geq 6^2 - 1 \). Combining this result with other existing results, \( H(2) \geq 2^2 \), \( H(4) \geq 4^2 - 1 \), and that for odd order systems, a conjecture is posed for Hilbert’s 16th problem: \( H(n) \geq n^2 \) or \( n^2 - 1 \).