Let us say that two sequences of pairwise distinct reals \( \ldots, a_1, a_2, \ldots \) and \( \ldots, b_1, b_2, \ldots \) defined on the same set \( S \) (which can be finite, or equal to \( \mathbb{N} \) or \( \mathbb{Z} \)) are equivalent if for all \( i, j \in S \) we have \( a_i < a_j \) if and only if \( b_i < b_j \). An equivalence class of sequences on \( S \) will be called an \((S-)\)permutation. An \( S \)-permutation can be also interpreted as a linear ordering of \( S \). A permutation \( a \) having a representative \( a = \ldots a_1, a_2, \ldots \) is called \( t \)-periodic if for all \( i, j \) such that \( i, j, i+t, j+t \in S \) we have \( a_i < a_j \) if and only if \( a_{i+t} < a_{j+t} \). An \( \mathbb{N} \)-permutation is called \emph{ultimately} \( t \)-periodic if the periodicity property holds for all \( i, j \geq n_0 \) for some \( n_0 \).

Surprisingly, for all \( t \geq 2 \) there exist infinitely many \( t \)-periodic \( \mathbb{Z} \)-permutations. We characterize them and give a way to code each of them.

Then we define complexity \( f_\pi(n) \) of a permutation \( \pi \) as the number of permutations (i.e., equivalence classes) \( \pi_k, \pi_{k+1}, \ldots, \pi_{k+n-1} \). Analogously to the subword complexity of words, this function is non-decreasing, and we have:

**Theorem 1** Let \( \pi \) be a \( \mathbb{Z} \) (\( \mathbb{N} \)-)permutation; then \( f_\pi(n) \leq C \) if and only if \( \pi \) is periodic (ultimately periodic).

However, other properties of subword complexity cannot be directly extended to complexity of permutations: in particular, one-sided and two-sided infinite permutations have different minimal complexity.

**Theorem 2** For each unbounded growing function \( g(n) \) there exists a not ultimately periodic \( \mathbb{N} \)-permutation \( \pi \) with \( f_\pi(n) \leq g(n) \) for all \( n \geq n_0 \). On the other hand, for each non-periodic \( \mathbb{Z} \)-permutation \( \pi \) we have \( f_\pi(n) \geq n - C \) for some constant \( C \) which can be arbitrarily large.

This is a joint work with D. G. Fon-Der-Flaass.