Let $G$ be a locally compact group, and let $\mu$ be a probability measure on $G$. Then a function $\phi \in L^\infty(G)$ is said to be $\mu$-harmonic if $\mu * \phi = \phi$. The $\mu$-harmonic functions do not form a von Neumann subalgebra of $L^\infty(G)$, but can be equipped with a product turning them into a von Neumann algebra in its own right. Dual to this situation, for a continuous, positive definite function $\sigma$ on $G$ with $\sigma(1) = 1$, A. T.-M. Lau and C.-H. Chu called an element $x$ of the group von Neumann algebra $VN(G)$ of $G$, $\sigma$-harmonic if $\sigma \cdot x = x$. Interestingly, the collection of all $\sigma$-harmonic elements is a von Neumann subalgebra of $VN(G)$.

Recently, W. Jaworski and M. Neufang extended the notion of a harmonic function to that of a harmonic operators. In this talk, which is based on joint work with Neufang, we develop a theory of harmonic operators from the dual perspective, thus extending Lau’s and Chu’s approach.