It is well known that the only proper non-trivial norm-closed algebraic ideal in the algebra $L(X)$ for $X = \ell_p$ ($1 \leq p < \infty$) or $X = c_0$ is the ideal of compact operators. The next natural question is to describe all closed ideals of $L(\ell_p \oplus \ell_q)$ for $1 \leq p, q < \infty$, $p \neq q$, or, equivalently, the closed ideals in $L(\ell_p, \ell_q)$ for $p < q$. We show that for $1 < p < 2 < q < \infty$ there are at least four distinct proper closed ideals in $L(\ell_p, \ell_q)$, including one that has not been studied before.

This is a joint work with B. Sari, Th. Schlumprecht and N. Tomczak-Jaegermann.