The asymptotic theory of infinite dimensional Banach spaces, developed by Maurey, Milman and Tomczak-Jaegermann, is concerned with the structure of infinite dimensional Banach spaces manifested in the finite-dimensional subspaces that appear everywhere far away in the space. The class of spaces that have a simple asymptotic structure, in the sense that we can find a $1 \leq p \leq \infty$ such that all such finite-dimensional subspaces as before are essentially $l_p^n$'s, are of special interest and they are called asymptotic-$l_p$ spaces.

We prove that if a Banach space is saturated with infinite dimensional subspaces in which all special $n$-tuples of vectors are equivalent, uniformly in $n$, then the space contains asymptotic-$l_p$ subspaces, for some $1 \leq p \leq \infty$. The proof reflects a technique used by Maurey in the context of unconditional basic sequence problem and extends a result by Figiel, Frankiewicz, Komorowski and Ryll-Nardzewski.