Littlewood, in his 1968 monograph “Some Problems in Real and Complex Analysis”, poses the following research problem, which appears to still be open:

“If the \( n_m \) are integral and all different, what is the lower bound on the number of real zeros of

\[
\sum_{m=1}^{N} \cos(n_m \theta)
\]

Possibly \( N - 1 \), or not much less.”

No progress appears to have been made on this in the last half century. Until now!