## Mathematics Education

(Org: C. Kieran (UQAM) and M. Santillana (UPN))

## TENOCH CEDILLO, Universidad Pedagógica Nacional

The potential of CAS in promoting changes in teachers' mathematical knowledge, practices and conceptions
This presentation will discuss a study aimed at providing plausible answers to the following questions:

- In which ways do middle school mathematics teachers take advantage of the algebraic transformation facilities offered by a CAS?
- In which ways does the use of CAS influence teachers' mathematical knowledge?
- In which ways do mathematics teachers change their practices as a result of using CAS in their teaching?

From the beginning of the study the teachers seemed to be convinced of the potential of CAS facilities to stimulate their students to explore, put forward and test mathematical conjectures. At this point it seemed interesting for the aims of this study to investigate why they also seemed to be reluctant to use those CAS facilities to carry out algebraic transformations. During the interviews it was found that teachers' reluctance was due to the value they gave to students' learning of algebraic transformation rules by paper-and-pencil techniques. However, they acknowledged that most of their students learn the algebraic algorithms meaninglessly.
I will discuss how teachers changed their view during the study and found how to take advantage of CAS in order to help their students learn in a more meaningful way such notions as how to simplify similar terms within an algebraic expression, the laws of exponents, and some non-conventional strategies to factor polynomial expressions "without making their students become key pushers".

ALEJANDRO DIAZ-BARRIGA, Universidad Nacional Autónoma de México, Instituto de Matemáticas, Circuito Exterior de Ciudad Universitaria, México, DF, CP 04510
A mathematician's perspective on teacher training in Mexico
Teacher training is done in Mexico at schools that are exclusively devoted to this aim, the "Escuelas Normales" and "Normales Superiores", depending on the level: the former for primary school (ages 6-12), the latter for secondary school (ages 1215). Mathematics teachers of the baccalaureate level (ages $15-18$ ) do not usually receive a pedagogical training but have degrees in subjects related to mathematics. Generally speaking, primary school pre-service teachers do at most one course in mathematics, and those who will become mathematics teachers in secondary school do two or three. There is an agreement among mathematicians that teacher training (especially future maths teachers') lacks mathematics.
In this paper we will expand on this subject, stressing the importance of the manner in which mathematics should be taught. We will give some examples of this, particularly for the secondary and baccalaureate levels.
The session will be shared with Silvia Alatorre (Universidad Pedagógica Nacional).

## OLIMPIA FIGUERAS, CINVESTAV

FLORENCE GLANFIELD, College of Education, University of Saskatchewan, 28 Campus Drive, Saskatoon, SK S7N 0X1, Canada
A Reflection about Mathematics Teacher Education Programs in Canada
In this session I will first start by providing an overview of mathematics teacher preservice programs in Canada. These programs differ for those who will be elementary mathematics teachers and for those who will be secondary mathematics teachers. In almost all teacher education programs in Canada, preservice teachers are required to take at least one mathematics content course as well as a mathematics 'methods' course (a course that teaches you how to teach mathematics). Mathematics educators in Canada ask the following questions about the preservice programs: Are these requirements sufficient to become a mathematics teacher? Should preservice teachers take more mathematics courses? What mathematics should preservice teachers know? In my talk I will propose the question, "Are these the questions that mathematics educators should be asking?" In my experience and research many elementary (and some secondary) preservice teachers do not see 'themselves' in mathematics; they come to preservice education courses with certain understandings and perspectives about mathematics and what it means to teach mathematics. These understandings and perspectives are framed by the experiences that preservice teachers have had prior to entering and while enrolled in their teacher education programs. Perhaps the questions that mathematics educators should be asking are not related to the content. . . but related to what we do in our programs to 'shift' the understandings and perspectives of preservice teachers.

FERNANDO HITT, Université du Québec à Montréal, Département de Mathématiques, Case Postale 8888, Succursale Centre Ville, Montréal, PQ, H3C 3P8, Canada
Reflexión sobre diferentes acercamientos en la enseñanza de las matemáticas en ambientes tecnológicos
Los profesores de matemáticas frente al uso de la tecnología en los procesos de enseñanza tienen, en general, posiciones extremas; una que va en contra del uso de la tecnología porque el profesor considera que su uso inhibe el aprendizaje y, la otra, con un entusiasmo desbordado, que la tecnología juega un papel fundamental en el aprendizaje de las matemáticas. Nos podemos preguntar cuál es la influencia que tienen los investigadores en la promoción implícita o explícita de estas posiciones. Quisiera, en esta exposición, analizar algunas investigaciones realizadas en ambientes tecnológicos en donde podríamos encontrar indicadores que nos permitan tener una posición crítica y reflexiva sobre el uso de tecnología, tanto en el terreno de la investigación sobre el estudio de fenómenos ligados al aprendizaje de las matemáticas como en su uso en los procesos de enseñanza.

## BERNARD R. HODGSON, Département de mathématiques et de statistique, Université Laval, Québec G1K 7P4, Canada

 The mathematical education of primary and secondary school teachers: the experience of Université LavalIn most Canadian universities, the education of pre-university teachers represents an important component of the task of the mathematics department, at least in terms of the number of students involved. Such is the case at Université Laval, where the Department of Mathematics and Statistics is responsible for several courses offered in the context of the pre-service education of teachers. These include two compulsory math courses given to prospective primary school teachers, and seven courses specific to prospective secondary school teachers.
The aim of this talk is to describe briefly the general framework of the mathematical education of school teachers at Université Laval and to discuss the main themes around which the content of the mathematics courses is articulated. Examples will be given of the mathematical topics presented in these courses; some of the pedagogical approaches will also be discussed.
Comments about both the departmental support of these activities, and the collaboration with colleagues from the Faculty of Education and from other departments of the Faculty of Sciences and Engineering, will be presented. A brief comparison will be made with other Canadian universities.

CAROLYN KIERAN, Département de mathématiques, Université du Québec à Montréal, CP 8888, succ. Centre-Ville, Montréal, QC H3C 3P8
Using Computer Algebra Systems (CAS) in Teaching High School Mathematics: Two Research-Based Examples from Classroom Practice

The integration of new technologies in mathematics education has been an ongoing issue for the last two decades. Teachers and teacher educators are struggling with questions regarding the use of technological tools and their relation to required paper-and-pencil skills. The original optimism regarding the benefits of technology, which would allow a focus on conceptual understanding at the expense of calculation techniques, has become quite nuanced. This presentation will address the dialectical relation between theoretical thinking and technique, as they co-emerge in a combined computer algebra and paper-and-pencil environment. More particularly, it will focus on two Grade 10 teaching experiments involving CAS technology: the first one on equivalence, equality, and equation; the second one on generalizing and proving within factoring. Attention will be given to the nature of the tasks in which the students engaged and to students' ways of thinking within these tasks. Even though the topics are quite different, findings indicate the importance of the co-emergence of theory and technique in both cases.

## LUIS RADFORD, Ecole des sciences de l'éducation, Université Laurentienne, Sudbury, Ontario, Canada, P3E 2C6 The Investigation of Motion and its Symbolic Mathematical Expression

Generally speaking perceptual activity, gestures, concrete actions, and natural language provide one with the basic resources to achieve a certain understanding of motion. However, as the studies conducted in the late Middle Ages suggest, the mathematical investigation of motion rests on a process of idealization achieved through the use of signs. This is why a mathematical investigation of motion requires not only the overcoming of the concrete experience and its intuitive, phenomenological key concepts (e.g. space, time, velocity), but also the understanding of new subtle concomitant forms of mathematical symbolization. In this presentation we pay attention to this idealization considered as a dialectic process between concepts and signs. We analyze some classroom excerpts that point to some of the students' difficulties in their attempt to understand and make sense of motion and its symbolic mathematical expression. It is suggested that rather than merely "representing" motion, algebraic symbolism (in its graphical or formulaic form) is an artifact. Algebraic symbolism mediates new ontogenetic ways of reflecting about the world that emphasize certain qualitative and quantitative relationships and leads to specific cultural conceptions of space, time and motion.

## ARTURO RAMIREZ, Centro de Investigación en Matemáticas, A.P. 402, Guanajuato, Gto., CP 3600, México Una propuesta didáctica usando geometría dinámica

En esta presentación mostraremos cómo motivar el teorema de la recta de Simson utilizando para ello el GeoLab, un laboratorio de geometría dinámica. Mostraremos primero cómo, utilizando un método de Monte Carlo podemos conjeturar el teorema de Simson: "Dado un triángulo arbitrario y un punto en su circuncírculo, las proyecciones de dicho punto a los lados del triángulo están alineadas", y recíprocamente "Si las proyecciones de un punto a los lados de un triángulo están alineadas, entonces el punto está en el círcuncírculo del triángulo".
Tenemos entonces que dado un triángulo, a cada punto de su circuncírculo se le puede asociar una recta, su recta de Simson. Mostraremos luego que la envolvente de la recta de Simson, conforme el punto se mueve en el circuncírculo es una deltoide que es tangente al triángulo dado.
Finalmente, construiremos el triángulo de Morley del triángulo y veremos cómo está relacionado con la deltoide mencionada en el párrafo anterior.
Si el tiempo lo permite veremos algunos otros teoremas geométricos difíciles de visualizar sin la ayuda de GeoLab.

MARCELA SANTILLAN, Universidad Pedagogica Nacional

THOMAS STEINKE, Ottawa-Carleton Catholic School Board, Ottawa, Ontario
From Toys to Tools: Reflections of a Math Consultant on Implementing Technology in K-12 Math in Ottawa and Ontario
Tom will share his reflections on his school board and Ontario, province-wide experiences related to the implementation of technology in K-12 mathematics in policy, teacher professional learning, and resource development. The experiences relate predominantly to the challenges and opportunities related to the insertion of CAS (TI 89), dynamic statistics software (Fathom and TinkerPlots), dynamic geometry software (GSP4) in our provincial mathematics policy documents. Some recent hopeful and innovative models for professional learning that focuses the effective use of technology to support improved student learning will be highlighted.

