The purpose of this vignette is to share how the investigation of canonical matrix forms can be used to highlight a rich variety of mathematical theory building philosophies and techniques at a relatively early stage of the undergraduate mathematics curriculum. Focusing on the matrix version instead of the equivalent linear transformation version helps make this topic accessible in a second course in linear algebra.

Taking a problem-first approach as opposed to the traditional theory-then-applications approach allows the students to see theory as arising naturally out of the study of problems. Furthermore, carrying the study of a single problem over an entire unit (perhaps three to four weeks in duration) helps students appreciate the process of mathematical investigation as a sustained, multifaceted endeavour, as compared to the solve-this-homework-problem-then-on-to-the-next-one nature of the typical activities assigned in lower level undergraduate mathematics courses.

The exposition of canonical forms given in the Outline section of this vignette is in no way novel. But what I believe is of interest is the variety of problem-solving philosophies and heuristics that can be exposed through this topic, as well as the pedagogical choice to bring these to the fore while allowing examples and discovery to lead to development of theory.