Mathematics for Elementary Teaching[1]

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Abstract

Research-based changes in elementary mathematics teaching and learning, as described for example by the NCTM *Standards* (2000), place significant demands on teacher capacity. Elementary (K-8) teachers in particular are often trained as generalists, yet *reform-based* or *Standards-based* classroom teaching requires a deep and specific body of mathematical knowledge in its own right. This report focuses on examining such a body of knowledge, as it is enacted by teachers and unpacked by the broader mathematics-education community with a specific focus on the mathematical elements. The report concludes with recommendations for significant improvements to specialised mathematical preparation for Canadian elementary teachers.

Rationale

Mathematics reform describes classroom learning processes that typically shift the instructional focus from one in which the teacher is presenting precedures, to having students actively engaged in developing their own understandings (NCTM, 2000). Part of this reform involves the classroom processes in which the teacher supports students' learning thorugh engagement in problem solving tasks. Students develop, represent, and justify their mathematical and strategic thinking, deepening their conceptual understanding as they do so. In support of these learning processes, the role of the teacher

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becomes one of a "mathematical mediator" who supports, probes, and negotiates valid mathematical thinking. Through questioning, the teacher aims to illuminate and clarify student thinking, and helps students identify any errors in reasoning while making sense of alternate strategies.

This type of classroom environment requires teachers to understand mathematics differently from the way they typically learned it - more deeply, flexibly, and conceptually. Negotiating varying student solutions and pinpointing reasoning errors, setting new tasks to address misconceptions, analyzing various models and using them to justify the development of generalisations, and supporting the making of connections within mathematics requires particularly deep and specialised understanding on the part of the teacher (Ball & Bass, 2003). Undergraduate mathematics courses may not intentionally develop a deep and flexible understanding of the elementary mathematics of classrooms, and professional development opportunities rarely focus on understanding mathematics for teaching.

We believe that the creation of a theory of mathematics for teaching, and strategies for supporting teachers who wish to expand their understanding of mathematics for teaching, need to include the voices of teachers. Existing definitions of mathematics for teaching (eg. Adler & Davis, 2006; Hill, Rowan & Ball, 2005) presume that testable and certifiable content available from universities' on-hand human resources offer a viable starting point. Yet testable mathematics relentlessly moves toward technical details such as vocabulary and rules at the expense of rich multi-representational conceptualizations organized for enactment in classrooms. Attending to appropriate conceptual understanding of elementary content as needed for effective teaching is of vital importance to teacher development at all levels.

Background

The notion of *pedagogical content knowledge* described by Shulman (1986) suggests that there is subject-matter-specific professional knowledge which bridges content knowledge and the practice of teaching (Ball, Thames & Phelps, 2008). However the view that there is a body of subject specific knowledge needed which is specialised for teaching is not universally endorsed, and in fact Ball and her colleagues state that "The prevailing view is that teachers need to know whatever mathematics is in the curriculum plus some additional number of years of further study in college mathematics" (2008, p. 394). An alternate view is that teachers need to know the curriculum, plus some amount of pedagogical content knowledge (Ibid.). "Little progress has been made toward a consensus on what teachers need to know." (Davis & Simmt, 2006, p. 294). Recently, researchers have argued for the existence of a distinct body of mathematical knowledge particular to the work of elementary teaching (Ball et al, 2008; Davis & Simmt, 2006; Kajander, 2007; In revision). Evidence has been suggested that "teaching may require a specialised form of pure subject matter knowledge – "pure" because it is not mixed with knowledge of students or pedagogy and is thus distinct from the pedagogical content knowledge identified by Shulman and his colleagues and "specialized" because it is not needed or used in settings other than mathematics teaching." (Ball et al, 2008, p. 396). For example, deciding whether a procedure would work in general, determining the validity of a mathematical argument, or selecting a mathematically appropriate representation requires mathematical knowledge and skill, not knowledge of students or teaching (Ball et al, 2008).

Our sense of the type of content knowledge needed for teaching impacts decisions about programs in significant ways. For example, if "the type of mathematical knowledge needed for teaching is basically the same as general mathematical ability, then discriminating professional development opportunities would be unnecessary." (Ibid, p. 399). On the other hand, if we believe that "The mathematical demands of teaching require specialized mathematical knowledge not needed in other settings" (Ibid., p. 401), then more specialised opportunities to learn mathematics *must* be provided to teachers. Hence the preparatory research for this working group began with the goal of examining the mathematical knowledge and thinking of experienced classroom practitioners as they went about the business of teaching elementary classroom mathematics, in order to further our understanding of the nature of such knowledge.

Preparatory Research

Introduction and Goals

We believe that a needs-based (or opportunity-based) description of mathematics for teaching that could then be operationalized by professional development processes that work for teachers is required. Teachers cannot describe fully the nature of something they do not possess; nevertheless, no one but teachers can identify the nature of the mathematics that is particular to their needs as mathematics teachers. A teacher-generated conception of mathematics for teaching is a goal that requires mathematicians and mathematics educators to listen to teachers while they assist those teachers to come to terms with their needs for their further mathematical development. In our ongoing research (Kajander & Mason, 2007; Kajander, submitted; Jarvis, in progress), we have been able to support teachers in describing their perceptions of the mathematics they want to be able to teach. As well, we have found that teachers and mathematics educators can collaboratively interrogate the nature of the mathematics the teachers feel they want to understand better in order to teach in an effective manner. The goal cannot be to develop yet another way to portray the deficits in teachers' understandings or another unrealistic list of university-level content; rather, teachers need to be supported to develop the understandings of mathematics as needed for teaching that they are ready to learn next. If we amplify, rather than judge, the voices of teachers who acknowledge they want to know more, a conception of mathematics for teaching can be developed that lends itself to helping teachers to pursue greater understandings. Such a conception is likely to be: developmental, acknowledging that needs will vary across differing stages of teachers' careers; contextual, acknowledging the immediacy of teachers' needs as experienced within particular classrooms; and strength-oriented, building from teachers' existing knowledge of mathematics as they teach it, rather than redressing deficits that emphasize the math that teachers do not yet know.

The pre-conference research incorporated an instrument for characterizing upper elementary teachers' knowledge of and beliefs about the mathematics they teach which enabled both preservice and in-service teachers to characterize and act to develop their understandings of and beliefs about mathematics for teaching (Kajander, 2007; in revision). As well, the progress of a professional development initiative for teachers working in the elementary to secondary transition years will be examined (Jarvis, In progress).

Data from teacher journaling, classroom observations, focus groups, and semi-structured interviews was used to bring to light and interrogate the teachers' perspectives of the mathematics for teaching they know, they want to know, and they are coming to know. For the *Forum*, we wished to include a specific focus on a teacher-sensitive conception of mathematics for teaching, as enacted in their classrooms and developed through their participation in professional learning opportunities.

Pre-conference Data Collection, Results, and Analysis

In Ann's project, a group of about eight elementary teachers were invited to participate in half-day focus-group meetings which took place approximately bi-monthly during the two years prior to the *Forum*. Although there were some changes in the composition of the group from one year to the next, five teachers remained as active participants during both years of the project, and four of these attended the *Forum* as working group participants. The purpose of these sessions was to collect data *from the teachers' viewpoints*, on their daily classroom experiences and interactions with students which involved the teachers' mathematical conceptions. All of these teachers were known to Ann prior to this research, having been involved in previous research and/or professional development activities with her, and all four teachers attending the *Forum* had also taught elementary teacher-candidates at some point at Lakehead University's Faculty of Education. Classroom observations of most of the teachers in the research group supported our belief that these were strong and highly dedicated teachers.

As well as from transcriptions of these focus group meetings, data was collected via teacher journaling, emails, field-notes from phone conversations, classroom observations, and individual meetings with participants which were recorded and transcribed. The researcher took the stance of a participant-observer, asking questions, responding, and at times, fully engaging in the dialog at the meetings. One teacher in particular was grateful for the opportunity for personal professional development during individual meetings, but the rest of the participants met only as a group. During most of the focus group meetings, the on-going dialog, while at times in response to particular prompts or activities posed by the researcher, was participant-driven.

Davis and Simmt claim that "… *for teachers*, knowledge of established mathematics is inseparable from knowledge of how mathematics is established" (2006, p. 297). Early journal entries written by the teachers in this project revealed the difficulties experienced by participants in separating or isolating what they needed to know about mathematics from what they needed to know about students and their learning, teaching strategies, and the curriculum. However, a number of examples certainly supported the claim that

> It is significant mathematical work that a teacher does when preparing for and coordinating a class discussion about their [students'] solutions to a problem in order to make mathematical ideas and strategies explicit. Such work requires teachers … to make sense of different strategies they may not have seen before, discern the mathematical details inherent in the solution, see mathematical relationships between solutions, and notice the solutions with strategies that would work for all cases. Such sense making requires teachers to deconstruct their own mathematics knowledge to make visible mathematical ideas and strategies in multiple forms (e.g., concrete, semi-concrete, abstract). (Kieran, Kubota-Zarivnij & Mason, 2008, p. 44).

A number of classroom examples were offered by the teachers during the focus-group meetings which indicated the need for teachers to have both "conceptual sophistication" (Davis & Simmt, 2006, p. 293) as well as a mathematical understanding which was "qualitatively different" (p. 294) from the knowledge held by students or by other more general knowers of mathematics. While the teachers found it hard to describe the mathematics they were using without referring to particular classroom examples, all participants were adamant that the understandings they needed included those that were specialised in nature. One teacher-participant summed up the struggle to define such knowledge by saying that what teachers needed to know about mathematics that was particular to teaching were "ways to bridge the gap between the concrete and the abstract, and the interconnections between one idea and another ... so that the ideas are reasonable". They also emphasized that they needed to be able to discern where a child was in their understanding, and come up with appropriate mathematical ideas to help them make the connection to the next idea: "you need to know about the interconnectedness of concepts … and to recognize the significance of what the kids are saying", and "you need to know how to get to the new idea from what the students know".

During the months directly preceding the *Forum*, particular activities for the Working Group were field tested during the focus-group sessions, and revised to be used during the actual Working Group. For example, items were selected by consensus of the teacher-group from five different instruments currently in use in the field to assess teachers' knowledge of mathematics. The instruments ranged from research instruments for both preservice and in-service teachers, to entrance and certification tests for teachers. These sample items (two from each instrument) were subsequently used as part of an activity during one of the introductory Working Group sessions, for participants to examine and discuss samples of the range of interpretations of knowledge of mathematics thought by researches and teacher-developers to be required by teachers. These varied items illustrated concretely to us the claim that "little progress has been made toward a consensus on the question of what teachers need to know" (Davis & Simmt, 2006, p. 294).

In addition to the focus-group with in-service teachers, a large sample (over 400) of preservice teachers has been studied with respect to their mathematical understandings at the beginning and end of their mathematics methods course. This work aligns with other studies which show that conceptual understandings of elementary mathematics tend to be critically weak for many teachers entering an elementary teacher education program, but do increase significantly during an appropriate methods course (Kajander, 2007; in revision). As well evidence indicates that greater gains in conceptual understanding as well as better development of knowledge of mathematics for teaching are achieved if more time is spent on the kind of specialised content as needed for teaching (Kajander, in revision).

In the year leading up to the CMEF event in Vancouver, Jarvis followed the progress of a local mathematics coordinator as she continued to implement a "family of schools" approach to professional development and learning within the local district school board. To better understand this Grades 7-10 initiative, interviews were conducted with the original coordinator who had set in motion the first "family of schools" group (i.e., secondary school mathematics teachers meeting with their elementary "feeder school" colleagues), the coordinator who was responsible for scaling up the initiative within the board, two elementary school teachers, and one secondary school mathematics teacher. Jarvis also attended the closing event for the year-long series of "family of schools" meetings, a day in which the various groups shared their progress, analyzed elementary/secondary student work samples, and debriefed on the entire experience in both their cross-panel groups and also with the entire group of teachers involved.

Although cross-panel initiatives can be difficult to organize and implement in terms of logistics, curriculum content focus, and general communication, the interviews revealed an overall positive experience. As Suurtamm and Graves (2007) reported in their large study of curriculum implementation in Intermediate mathematics classrooms in Ontario, cross-panel initiatives often reveal inconsistent views of mathematics pedagogy:

> The mathematics leaders reported that when groups of teachers from different panels have the opportunity to get together, there are often very different views of teaching and learning expressed. In addition, their discussions revealed that the positions of elementary and secondary teachers are not necessarily consistent within each panel. In one group, the elementary teachers were noted as being more traditional than the secondary, whereas another group of mathematics leaders described the secondary teachers as more traditional. (p. 22)

In the Jarvis research interviews, the local coordinator explained how the "family of schools" approach involved a shared problem solving focus, curriculum analysis, and, perhaps most importantly, cross-panel visits to classrooms within the board. She notes:

> There was always a big accountability in everything we did, so when they were doing the lesson studies and developing the lessons, they all had to go out and try the lesson in their class and then bring it back. . . . The Grade 8 and 9 teachers got to see how the Grade 7 teacher taught it, what kind of work the students were producing, the anticipated responses, where the problems were, and they got to see all three grades. . . . So they saw that progression and got to see some of the

> things that they could do back and forth. That was really key, getting to see each level—everyone watched a Grade 7, 8 and Grade 9 lesson on the same topic, where it was possible. The Grade 7, 8, and 9 don't all line up exactly so that the topic could flow, but we tried to find a similar topic that would have some relationship to what was taught in the elementary school.

In terms of the "mathematics for teaching elementary" focus of this working group report, these cross-panel classroom observations and subsequent opportunities for discussion about curriculum, teaching strategies, and student learning were critical for establishing a more consistent approach to teaching mathematics across school divisions.

In the several months prior to the *Forum*, a series of six Skype or video conference meetings took place with the Working Group leaders and (at times) various teachers from the focus groups.

Working Group Activities

Goals

Our idealized goal was for teachers, mathematics educators, and mathematicians to work together toward a shared vision and understanding of "mathematics for teaching" as it might originate from teachers' understandings and beliefs, and be generalized to a more complete conception which very much included teachers' voices. Specifically, we wanted to invite CMEF participants to respond to a conceptualization of knowledge for teaching drawn from teachers, commenting on its descriptive qualities and its generative possibilities (and its potential as a starting point for effective collaboration through professional development). Lastly, we hoped to construct a Canadian policy statement which might be helpful for mathematics educators and others charged with supporting teachers' mathematical development, by supporting their efforts and arguments toward improved opportunities for development of teacher knowledge of mathematics for teaching.

Sessions

Session 1 (2 hrs)

In the first session we focused on the key questions, "What is elementary Mathematics for Teaching? Does it even exist?" Three popular views were discussed, namely: (i) that one simply requires more university/college mathematics credits to be a better teacher; (ii) that one needs to simply develop and adopt the understandings required for a more problem-based approach in teaching and understanding; or, (iii) that there is an actual body of knowledge/understanding specific to teaching children mathematics that must be learned and practiced.

Participants were then invited to think about two specific mathematical examples chosen from the data generated during the teacher-focus-group sessions, namely the subtraction of negative integers, and the notion of discrete versus continuous data as it might apply to elementary curriculum. With partners or in small groups, WG participants were then asked to discuss the question, “What do teachers need to know about these topics in order to teach them effectively?” A lively discussion followed in which the whole group shared ideas related to specialised teacher knowledge, such as appropriate knowledge of multiple representations, visualization, higher level mathematical ideas associated with the given topics, grade level appropriateness, related technology, etc. The easily-reached consensus of the group was that while general mathematical background is helpful, and while an understanding of the strategies associated with problem-based learning allows for many of the reform-based modelling/representation approaches, there is definitely a need for some form of specialized mathematics for teaching knowledge by teachers in the elementary classroom. As one participant put it, “…the skill of being able to entice a student to explain his or her method of problem-solving and then to facilitate a more theoretical connection to the concept will trump knowing ‘a lot of math’ every time”. Participants felt that “teachers must be able to recognize the mathematical significance of their students’ work (correct and with errors) ‘on the fly’”, and “be able to use that information to determine what that student needs next to move forward with his/her thinking.” In order to do this, it was expressed that teachers need to know how to choose appropriate “models, alternate representations and approaches, and questions”. It was also felt that teachers need a “mastery of multiple conceptual representations to support the development of conceptual knowledge in others to the point that they are able to fluently generate examples for students to explore.”

In a second activity, participants were given colour-coded sheets with mathematics questions drawn from a variety of teacher knowledge assessment tools, to generate further small-group discussion. A template was used to comment on the various tools, and some small groups chose to use a continuum type of analysis for the samples, ordering them from more generally content-focused to more specialized. Again, this exercise was discussed in light of the larger question for this session as stated above.

For example, most participants felt that while the item

1. Order the following numbers from least to greatest.

   0, -5/3, 4.1, 7/13, -2.95, +5/8, 1.8, -1.1

[Source: OISE/UT mathematics practice items for Intermediate mathematics examination for admission to Additional Qualification courses, downloaded from the web spring 2009]

would require conceptual understanding to correctly answer, it did not require specialized understanding, while the items

2. Write a story problem that could be used to motivate each calculation.

   a) ¾ ÷ ½
   story problem:

   b) ¾ ÷ 2
   story problem:

3. The “traditional fraction multiplication procedure” tells us to multiply the numerators and multiply the denominators of two fractions to get the product. Use the example ¾ x ⅝ with

> a visual model (diagram) to conceptually justify the steps in this particular procedure. Use pictures and words to explain fully what is happening in the procedure and why it makes sense related to the diagram. State the answer in relation to the diagram.
>
> [Source : Lakehead University junior-intermediate methods course final Math-for-Teaching content examination, spring 2009]

would require a more specialized knowledge.

Session 2 (1.5 hours)

For Session 2, we divided up into two groups with Ann hosting a discussion around pre-service teacher education and Dan hosting a discussion around in-service teacher education. Two main questions provided direction for these small group discussions: “What strategies are in place at your institution to support teacher growth in mathematics as needed for teaching?” and, “In your opinion, what should be in place?” During her session, Ann shared a bit of her research related to the significant difference in mathematical understanding for teaching that is possible to develop when teacher candidates are offered more time to learn appropriate mathematics for teaching. Dan shared a brief report and some teacher/coordinator quotations from the cross-panel professional development that had formed the basis of his recent case study research. Participants in both groups shared updates regarding the types of B.Ed. programs in which they are involved (e.g., number of years/hours per course, available resources, textbooks used, assessment tools). Much of the thinking shared in this session would help to later develop the national policy statement regarding mathematics for teaching.

Session 3 (joint with *Problem-Solving in Elementary Mathematics* working group, 1.75 hrs)

In the third session, we partnered with the Working Group facilitated by Annie Savard and included Immaculate Kizito-Namukasa, Elena Arkhipova, Anna Sanalitro, and Irene Percival. Each group presented an overview of their group activities and thinking to date. There was good discussion during/following these presentations in terms of the need for problem-based learning approaches, the need for the recognition by teachers of student misconceptions and the need for supporting of alternate solution strategies.

Session 4 (1.75 hrs)

In the fourth and final Working Group session, participants were asked to carefully examine the main question, “What do we recommend be provided in Canada for elementary teachers (both pre-service and in-service) to support the development of appropriate teacher knowledge of mathematics?” Brainstorming and discussion produced ideas which ranged from changing the entire structure of B.Ed. programs in Canada to more minor modifications in course organisation, content, delivery and lengths. A “revolution” was one recommendation: “Having a one-year (or even two-year) boot camp after a degree is counter-productive. There needs to be a degree in Elementary Education, in which about half the time is spent on learning content and the rest on psychology (child development) and methods. This can come about only as a revolution, i.e. systematic change".

Understanding that existing programs vary greatly in their structure and emphases across the country, one important point that the group spent a considerable amount of time discussing was the number of hours that should be allotted for specialized mathematics content and pedagogy course offerings. While there was general agreement that approximately 100 hours of specific mathematics preparation for elementary teachers might be appropriate at the pre-service level (for example, two 50 hour courses, or three 36-hour courses, using typical course lengths at our various institutions), there were differing points of view as to whether *one* course should be recommended as an absolute minimum, with the 100 hours being recommended as more ideal. The more-ideal 100 hours was eventually the only recommendation placed in the Policy, and this recommendation does align well with the 115 hours recommended in the recent US report on the preparation of elementary teachers of mathematics from the National Council on Teacher Quality (2008).

Concerns were also expressed about the availability of appropriate mathematics in-service for teachers. In most jurisdictions, teachers are not required or even rewarded appropriately for such professional development work, and in many cases it is not even available. Worse still, some provinces have structures that are counter-productive. In Ontario for example, certain Additional Qualifications (AQ) professional development courses for teachers require a number of university mathematics credits be taken before teachers can enrol in the AQ courses! Such a requirement is a manifestation of the previously-prevalent notion that teachers simply need more mathematics courses of an unspecialised nature, and virtually guarantees that a math-anxious or even a more "typical" elementary teacher will be discouraged from furthering their more appropriate mathematical content knowledge for teaching.

One of our goals for the conference was to produce, or at least work towards, a Canadian policy statement on mathematics content preparation for elementary teaching, and this writing was begun during, and continued following, the fourth session. In the days and weeks to follow, Ann circulated a number of drafts of the policy statement, inviting feedback from the Working Group members and revisiting the draft document accordingly. This Policy is provided at the end of the current report of our Working Group (see Appendix A). Since its completion, Working Group members have been vigorously engaged in circulating this Policy Statement as broadly as possible, and readers are invited and encouraged to do the same.

Discussion and Conclusions

During both the pre-conference research activities as well as the actual Working Group sessions, we were struck by the like-mindedness of the participants (teachers and researchers alike) as to both the existence of a specialized domain of mathematical knowledge required for elementary teaching, as well as the need for enhanced opportunities for teachers for its development.

The notion of a qualitatively different understanding as described earlier may also be helpful in our thinking; “whereas the work of research mathematicians might be described in terms of ‘compressing’ information into increasingly concise and powerful formulations, the work of teachers is more often just the opposite: teachers must be adept at prying apart concepts, making sense of the analogies, metaphors, images, and logical constructs that give shape to a mathematical construct” (Davis & Simmt, 2006, pp. 300-301). Such a conception, which might be thought of as intuitively opposed to the usual notion of mathematics which might involve abstractions and generalisations of an increasingly sophisticated nature, may not be comfortable to the broader mathematics community. In our informal discussions, examples of mathematical knowledge for teaching have at times been misunderstood by mathematicians as unnecessarily complicating matters – for both teachers and students. For this reason, it is also argued in the Policy that instructors of mathematics courses for elementary teachers must have some understanding of, and affinity for, the kinds of mathematical work that teachers do, and the kinds of mathematical knowledge that they need to do it. This is not to say that mathematics faculty should not teach such fundamental courses to teachers, but rather to emphasise the specialised nature of the content and the attention required to its development.

We are very grateful to the colleagues who commented on the draft when we (Ann) described and circulated the Policy statement at the 2009 *Canadian Mathematics Education Study Group Conference* (Kajander, accepted). While many colleagues were supportive of our aims, a few constructive comments deserve response. Firstly, we do not in any way want to preclude mathematics faculty from teaching mathematics for teaching courses. Rather we wish to underscore that the nature of the mathematics involved is indeed specialised, in both content and nuance, as described. Also it is important that mathematics faculties in general realise that such courses are foundational, and the choice of instructor is critical. Without a working knowledge of the issues, the philosophical nature of mathematics for teaching may indeed strike mathematicians as unnecessarily complicated (Kajander, in review). In fact, Moreira and David (2008) suggest that the values and forms of conceptualising objects in formal mathematics may differ from, and even collide with, the demands of classroom teaching practice. While many mathematicians are experienced in mathematics education issues and sensitive to the special needs of prospective elementary candidates, such experience is not gleaned without effort or intent.

Secondly, a one-page policy is, by its very nature, brief, and may indeed not address many of the related important issues. While we hope that the Policy will be useful to mathematics educators, professional mathematics organisations, school board personnel and others as they work towards and argue for more attention to professional development at all levels in mathematics for teaching, we also hope the existence of the Policy will encourage stakeholders to seek out more comprehensive statements such as the one written by a sub-committee of the Mathematics Education Forum of the Fields Institute (Sinclair, 2005).

We believe that only through vigorous promotion of the kinds of changes described in the Policy will we begin to do justice to teachers in terms of providing them with the capacity to do their job as it should be done and many would like to do. Only then will children more consistently have the opportunity to understand mathematics as a domain that makes sense, develops with logic, structure and predictability, and has inherent beauty and appeal.

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APPENDIX A

Policy Statement for Canadian Elementary (K-8) Teacher Mathematics Content Development

Teachers of elementary school mathematics believe that all of their students should succeed in mathematics, and now is the time to support such a stance. However, fundamental changes in mathematics teaching and learning place significant demands upon teacher capacity. Teachers need a specialized mathematical knowledge in order to enhance students' conceptual understanding as well as to support students' confidence in, as well as their capacity for, continuing to learn mathematics.

While it is essential that teachers have a deep, well interconnected, understanding of the concepts that students in elementary school will encounter, significant evidence indicates that there is a body of mathematical knowledge that is particular to the work of elementary teaching. Knowledge of how mathematical understanding may develop in children, as well as of the models, representations and practices that support students' mathematical development are essential. Teachers need to quickly recognize the mathematical significance of their students' thinking in order to determine and respond to misconceptions, and through appropriate questioning or provision of alternate tasks, move students forward. Appreciating and responding to alternative student approaches requires deep and flexible conceptual understanding on the part of teachers, as well as the ability to unpack students' thinking in order to recognize generalizable strategies or identify student misconceptions. Also needed is a deep and flexible understanding of operations, and the ability to make multiple connections to models, contexts, and essential mathematical activities such as reasoning and problem solving. This specialized knowledge does include highly *mathematical* components, such as the understanding of such multiple models and representations including appropriate use of manipulatives and technology, the mathematical knowledge needed to design and select classroom tasks useful in supporting student learning, as well as the mathematical insight required to make appropriate choices when anticipating and addressing student responses. Such fundamental aspects of effective elementary mathematics teaching need to be addressed prior to, or concurrent with, other teacher preparation courses.

Many elementary teacher candidates suffer from a lack of confidence and even anxiety about mathematics, often largely due to initially weak conceptual mathematical understanding. Confidence and a willingness to continue mathematical self-development can and should be supported in preservice teacher education programs. It is recommended that approximately 100 hours be provided for preservice teachers in specialized **mathematics knowledge for teaching** (for example, two 50-hour courses, or three 36-hour courses) which should *not* be replaced by more general undergraduate mathematics courses, as the content differs. Faculty members teaching these foundational courses should have specific mathematics education interests and background, which may require special attention when such courses are offered through mathematics departments. In addition, a similar amount of instruction in mathematics methods (content specific pedagogy) should be provided. These recommendations are particularly crucial for teachers of grades 5 to 8. Ideally, these recommendations should be further strengthened by appropriate teacher mathematics and mathematics education courses being taken over multiple years of an undergraduate degree program. Concerted efforts towards standardization of certification requirements of mathematical competence for teaching, as part of preservice program requirements across Canada, are highly recommended.

Both new and experienced classroom teachers should be encouraged to seek on-going professional development opportunities such as mentorships and mathematics-for-teaching courses, and such efforts should be supported and rewarded much more formally at the school, school board, and provincial levels. In particular, continuing education courses focused on mathematics-for-teaching should be provided for in-service teachers, and previous mathematical background, such as a prescribed number of general undergraduate mathematics courses, should *not* be used to preclude teachers from enrolling in such courses. The development and support of specialist mathematics teachers in grades 5 to 8 is strongly recommended.

In summary, elementary teachers require specialised mathematical knowledge for effective classroom teaching, and the development of such knowledge is a critical component of teacher competence as required to effectively support improved student learning and outcomes.